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Long Memory versus Structural Breaks in Modeling and Forecasting Realized Volatility

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Abstract

In this paper, we explore the possibilities of structural breaks in the realized volatility with the observed long-memory property for the Deutschemark/Dollar, Yen/Dollar and Yen/Deutschemark spot exchange rate realized volatility. The paper finds the substantial reduction of persistence of realized volatility after removing the breaks. Our VAR-RV-Break model provides the superior predictive ability compared to most of the forecasting models when the future break is known. The VAR-RV-I(*d*) long memory model, however, is still the best forecasting model even when the true financial volatility series are created by structural breaks with unknown break dates and size.

Keywords: Realize volatility; Exchange rate; Long memory; Structural break; Fractional integration; Volatility forecasting

JEL classification: C32, C52, C53, G10

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1. Introduction

Conditional volatility and correlation modeling has been one of the most important areas of research in empirical finance and time series econometrics for the past two decades. Asset return volatility and correlation, henceforth volatility, are especially central to finance, as they are key inputs for asset and derivatives pricing, portfolio allocation, and risk measurement. Although daily financial asset returns are approximately unpredictable, return volatility is time-varying but highly predicable with persistent dynamics.[1] Furthermore, the dynamics of volatility is well modeled as a long memory process. An inherent problem for measuring, modeling and forecasting conditional volatility is that the volatility is unobservable or latent, which implies modeling must be indirect. Typically, measurements of conditional volatility are from parametric methods, such as GARCH models or stochastic volatility models for the underlying returns. However, these parametric volatility models depend on specific distributional assumptions and are subject to misspecification problems.

Given the availability of intraday ultra-high-frequency price and quote data on assets, Andersen, Bollerslev, Diebold, and Labys (2003), henceforth ABDL, and Barndorff-Nielsen and Shephard (2001, 2002, 2004) introduced a consistent nonparametric estimate of the price volatility that has transpired over a given discrete interval, called realized volatility. They computed daily Deutschemark/Dollar, Yen/Dollar, and Deutschemark/Yen spot exchange rates realized volatilities simply by summing high-frequency finely sampled intraday squared and cross-products returns. By sampling intraday returns sufficiently frequently, the model-free realized volatility can be made arbitrarily close to underlying integrated volatility, the integral of instantaneous volatility over the interval of interest, which is a natural volatility measure.

[1] The findings suggest that volatility persistence is highly significant in daily data but will weaken as the data frequency decreases.

ABDL found logarithmic realized volatility could be modeled and accurately forecast using simple parametric fractionally integrated ARFIMA models. Their low-dimensional multivariate realized volatility model provided superior out-of-sample forecasts for both low-frequency and high-frequency movements in the realized volatilities compared to GARCH and related approaches. Many studies, however, have pointed out that observed long memory may not only be generated by linearly fractional integrated process but also by: (1) cross-sectional aggregation of stationary series (Granger and Ding 1996); (2) mixture of numerous heterogeneous short-run information arrivals (Andersen and Bollerslev 1997); (3) non-linear models, such as structural breaks (changes) or regime switches (Granger and Hyung 2004; Choi and Zivot 2006; Diebold and Inoue 2001). In particular, it has been conjectured that persistence of asset return volatility may be overstated with the presence of structural change.

In this paper, we focus on the possibilities of structural breaks in the realized volatility, with the observed long-memory property, for the Deutschemark/Dollar, Yen/Dollar and Yen/ Deutschemark spot exchange rate realized volatility from ABDL. First, we test for long memory and estimate long memory models for the realized volatility series. We find strong evidence of long memory property in exchange rate realized volatility. Second, we test for and estimate a multiple mean break model based on Bai and Perron (1998, 2003). We find several common structural breaks within the three series. Third, we exam the evidence for long memory in the break adjusted data. We find a substantial reduction of persistence in realized volatility after the removal of breaks. The evidence suggests that part of the long memory may be accounted for by the presence of structural breaks in the exchange rate volatility series.

Finally, we find that our VAR-RV-Break model provides competitive forecasts compared to most of the forecasting models considered by ABDL if future break dates and sizes are known. The VAR-RV-I(*d*) model, however, is still the best forecasting model even when the true financial volatility series are created by structural breaks and we have little knowledge about break dates and size.

The rest of the paper is organized as follows. Section 2 presents the long memory model and estimations. Section 3 presents empirical results using structural breaks model and examines the long memory estimations after adjusted breaks series. Section 4 reports the evaluation for forecasting. Section 5 concludes.

2. Realized Volatility and Long Memory Model

2.1. Realized Variance

ABDL utilized an empirical measure of daily return variability called realized volatility, which is easily computed from high-frequent intraday returns. By treating volatility as observed rather than latent, volatility modeling and forecasting using simple ARFIMA models is straightforward.

We assume that an arbitrage-free logarithmic price $p_t = \log(P_t)$ process can be expressed as a continuous-time diffusion process in terms of the following stochastic differential equation without a jump term,

$$dp_t = \mu_t dt + \sigma_t dW_t \quad (1)$$

where μ_t is the predictable drift coefficient, σ_t is the instantaneous volatility of the logarithmic price process, and W_t is a standard Brownian motion. We denote the daily continuously compounded return as

$$r_t = p_t - p_{t-1} = \int_{t-1}^{t} \mu_s ds + \int_{t-1}^{t} \sigma_s dW_s \quad (2)$$

where $\int_{t-1}^{t} \sigma_s dW_s$ is a local martingale, and we denote the corresponding integrated variance (IV_t) as

$$IV_t = \int_{t-1}^{t} \sigma_s^2 ds \tag{3}$$

This natural measure of the inherent return variability, however, is not directly observable. Realized variance (RV_t) is computed by simply summing cross-products of intraday returns,

$$RV_t \equiv \sum_{i=1}^{1/h} r_{t-1+ih}^{(h)} \cdot r_{t-1+ih}^{(h)'} \equiv R'_{t,h} R_{t,h} \approx IV_t \tag{4}$$

where $r_t^{(h)} \equiv p_t - p_{t-h}$ is the intraday return, $R'_{t,h} \equiv (r_{t-1+h}^{(h)}, r_{t-1+2h}^{(h)}, \ldots.., r_t^{(h)})$, h is sample frequency [2] and *1/h* is assumed to be an integer. ABDL showed that in the absence of measurement error in high frequency returns, realized variance is consistent for integrated variance as $h \to 0$. In practice, however, there is a lower bound on the sampling frequency because of market microstructure frictions features such as, discrete price, transactions costs, and bid-ask spreads at the very highest frequency.

2.2. Data

We use the same data as ABDL, which are spot exchange rates for the U.S. dollar, the Deutschemark, and the Japanese yen from December 1, 1986 through June 30, 1999.[3] Following ABDL, we choose equally-spaced thirty-minute[4] return to keep away from microstructure noise.[5]

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[2] For example of the 30-minute intraday sample frequency from a 24-hour trading day (1440 minutes), *h* is 30/1440=1/48. There are 48 intraday returns.

[3] The raw data include all interbank DM/$ and Yen/$ bid/ask quotes shown on the Reuters FX screen provided by Olsen & Associates. These three currencies were the most actively traded in the foreign exchange market during the sample period.

[4] Bandi and Russell (2003) suggested that sample horizon range from 5-minute to 30 minute interval is optimal as the minimization of the conditional mean-squared error of the realized volatility estimator.

Their realized variance construction process is as follows. We get thirty-minute prices from the linearly interpolated logarithmic average of the bid and ask quotes for the two ticks immediately before and after the thirty-minute time stamps over the global 24-hour trading day. Thirty-minute returns are obtained from the first difference of the logarithmic prices. We exclude all the returns from Friday 21:00 Greenwich Mean Time (GMT) to Sunday 21:00 GMT and certain holiday periods to avoid weekend and holiday effects. Our final data set consists of 3,045-days bivariate series of DM/$ and Yen/$ 30-minute returns over the sample period. The intraday return is denoted $r_t^{(h)}$, where $t = h, 2h, 3h, \ldots\ldots 47h, 1, 49h, \ldots\ldots, 3045$, where $h = 1/48 = 0.0208$.

As in equation (4), realized volatility for DM/$ and Yen/$ will be the diagonal elements of $R'_{t,h}R_{t,h}$. By absence of triangular arbitrage, the Yen/DM returns can be calculated directly from the difference between the DM/$ and Yen/$. Therefore, we get 3,045 observations of realized variance for three exchange rate series, as shown in Figure1. Figure 2 shows the realized volatilities, also called realized standard deviations, which are calculated from the square root of the realized variance. Both series show strong persistence and occasional clustering as well as possible jump patterns.

2.3. Realized Volatility Distributions

As shown in Table 1 and the left panel of Figure 3, the distributions of three realized volatility series are all right-skewed and fat-tailed. The distribution of logarithmic realized volatilities, however, are close to Gaussian as the logarithmic transformation reduces the impact of outliers. The kernel density estimates in the right panel of Figure 3 and the Q-Q plots in Figure

[5] The findings suggest that volatility measured at an interval shorter than 5-minute are cursed by spurious serial correlation due to nonsynchronous trading, discrete price observations, intraday periodic volatility pattern, and bid-ask spread.

4 provide strong evidence for the log-normality property for realized volatility. Last, the Ljung-Box statistics indicate strong serial correlation in all of the series.

2.4. Long Memory Model

Before conducting further modeling and forecasting, it is very important to determine whether the time series is stationary or not. However, the distinction between I(0) and I(1) for the conditional mean may be far too narrow. Long memory model that allows fractional orders of integration, I(*d*), provides more flexibility. For an I(0) process, shocks decay at an exponential rate; for an I(1) process, shocks have permanent effect; for an I(*d*) process, shocks dissipate at a slow hyperbolic rate. Long memory behavior in volatility has been well established, see for example, Ding, Granger, and Engle (1993), Baillie, Bollerslev and Mikkelsen (1996), and Andersen and Bollerslev (1997).

A time series process, y_t, with autocorrelation function ρ_k at lag *k*, is a long memory process when

$$\lim_{n\to\infty}\sum_{k=-n}^{n}|\rho_k|\to\infty \qquad (5)$$

The spectral density $f(\omega)$ tends to infinity at zero frequencies. In contrast, for a stationary process with short memory, the autocorrelation function is geometrically bounded, i.e. $|\rho_k|\le cm^{-k}$ with $0 < m < 1$. Granger and Joyeux (1980) and Hosking (1981) show that a long memory process for y_t can be modeled parametrically as a fractionally integrated process I(*d*), if

$$(1-L)^d(y_t-\mu)=\varepsilon_t \qquad (6)$$

where *L* denotes the lag operator, *d* is fractional difference parameter, μ is the unconditional mean of y_t, and ε_t is independent and identically distributed with zero mean and finite variance.

The fractional difference filter $(1-L)^d$ is defined as the binominal expansion

$$(1-L)^d = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + ... = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(k+1)\Gamma(-d)} \tag{7}$$

where $\Gamma(k+1)$ is the Gamma function. A more flexible process called the ARFIMA (*p, d, q*) model[6] allows $(1-L)^d(y_t - \mu)$ to be auto autocorrelated:

$$\phi(L)(1-L)^d(y_t - \mu) = \theta(L)\varepsilon_t \tag{8}$$

where $\phi(L)$ and $\theta(L)$ are autoregressive and moving average polynominals, respectively, with roots lie outside the unit circle. An ARFIMA process is non-stationary when $|d| > 0.5$ and stationary when $|d| < 0.5$. When $0 < d < 0.5$, y_t is called stationary long memory. When $-0.5 < d < 0$, y_t is called intermediate memory and antipersistent. When $d = 0$, it is simply short memory.

2.5. SEMIFAR Model

To allow for the data-driven distinction of long memory, short memory, stochastic trends, and deterministic trends without any prior knowledge, Beran and Ocker (2001) proposed a semiparametric fractional autoregressive (SEMIFAR) model

$$\phi(L)(1-L)^{\delta}((1-L)^m y_t - g(i_t)) = \varepsilon_t \tag{9}$$

where δ is the long memory parameter, and $g(i_t)$ is a smooth trend function on [0,1] with $i_t = t/T$. y_t must be differenced to achieve stationarity by parameter $d = \delta + m$. m determines whether the trend should be estimated from the original data (when $m = 0$) or the first difference

[6] To obtain a stationary process, y_t must be differenced *d* times. The parameter *d* determines the long-term behavior, whereas *p* and *q* affect the short-term properties.

(when m = 1). When $\delta > 0$, y_t is long memory. When $\delta < 0$, y_t is antipersistent. When $\delta = 0$, y_t has short memory.

2.6. Long Memory Estimation

According to the slow decay of autocorrelations in Figure 5, it is evident that the logarithmic realized volatility for the exchange rate series appears to have long memory dynamics. To estimate the long memory parameter *d*, we use the method of Geweke and Porter-Hudak (1983), henceforth GPH, based on the simple linear regression of the log periodogram on a deterministic regression

$$\ln[I(\omega_j)] = c - d\,\ln[4\sin^2(\omega_j/2)] + u_j, \qquad j = 1,\ldots..n \tag{10}$$

where $I(\omega_j) = (1/2\pi)\left|\sum_{i=1}^{T} y_t \exp(i\omega_j t)\right|^2$ is the periodogram at frequency $\omega_j = 2\pi j/T$. The window size *n* depends on the sample size *T*. The least squares estimator *d* will be asymptotically normal with variance $\pi^2/6n$. There are several other methods of testing long memory time series, and we also use them as a robustness check. For a detailed discussion of long memory testing methods, see Baillie (1996), and Robinson (1995).

The estimates of *d* for realized volatility are reported in Table 2, and the estimates of *d* for logarithmic realized volatility are reported in Table 3. Whether used nonparametric, parametric, or semiparametric methods, all of the estimates of *d* are in the range between 0.34 and 0.58, which confirms the long memory property in the (logarithmic) realized volatility.

3. Structural Break Model

3.1. Multiple Structural Break Model

It is well known that structural change and unit roots are easily confused (see Perron 1989; Zivot and Andrews 1992). Recently the confusion between long memory and structural change has been getting more and more attention. Granger and Ding (1996), Granger and Hyung (2004), and Choi and Zivot (2006) suggest that observed long memory property in the asset return volatility may be explained by the presence of structural breaks. To investigate this conjecture for realized volatility, we use the pure multiple mean break method proposed by Bai and Perron (1998, 2003), henceforth BP, to test this hypothesis. The *m* model (*m* + 1 regimes) is defined as

$$y_t = c_j + u_t, \quad t = T_{j-1} + 1, T_{j-1} + 2, ..., T_j \tag{11}$$

where j = 1, 2, …, m + 1, y_t is the logarithmic realized volatility, and c_j is the mean of the logarithmic realized volatility. The break points ($T_1, T_2, ..., T_m$) are treated as unknown. The error term u_t may be serial correlated and heteroskedastic. The estimation is based on the least-squares principle. The estimated break points ($\hat{T}_1, \hat{T}_2, ..., \hat{T}_m$) are obtained by solving $\underset{T_1,...,T_m}{\arg\min} S_T(T_1, T_2, ..., T_m)$ where

$$S_T(T_1, T_2, ..., T_m) = \sum_{j=1}^{m+1} \sum_{t=T_{i-1}+1}^{Ti} (y_t - c_j)^2 \tag{12}$$

Given the estimated break points, the corresponding estimates $\hat{c}_j(\hat{T}_1, \hat{T}_2, ..., \hat{T}_m)$ are obtained for each regime. We used several tests for structural change proposed in BP. Let $\sup F_T(l)$ denote the F statistic for the null of no structural breaks versus an alternative hypothesis containing an arbitrary number of breaks, and let *M* denote the maximum number of breaks allowed. We set *M* = 5. Define the double maximum statistic $UD_{\max} = \max_{1 \le l \le M} \sup F_T(l)$, and the weighted double max statistic $WD_{\max} = \max_{1 \le l \le M} w_l \sup F_T(l)$, where the marginal p-values are equal across values of l. The null hypothesis of both tests is no structural breaks against the alternative of an unknown number of breaks given some specific upper bound *M*. Sequential $\sup F_T(l+1|l)$ tests the null of *l* breaks versus the alternative $l+1$ breaks. To determine the number of breaks, we first use the $UD_{\max}$ and $WD_{\max}$ to determine if at least one break occurred. If there is evidence for structural change, we select the number of structural breaks using the $\sup F_T(l+1|l)$. To allow for a penalty factor for the increased dimension of a model, the above procedure may be complemented by selecting the number of breaks by minimizing a Bayesian Information Criterion (BIC) and a modified Schwarz Criterion (LWZ).

3.2. Multiple Structural Break Estimation

Table 4 displays the values of all the tests used to determine the number of breaks for the logarithmic realized volatility series. The *UD*max and *WD*max tests point to the presence of multiple breaks for all series. The $\sup F_T(l)$ tests reject the null hypothesis of no breaks versus the alternative of an unknown number of breaks for the all series. For DM/$, the $\sup F_T(l+1|l)$ is significant at 1% level when *l* = 4, which suggests 5 breaks. BIC suggests 5 breaks as well while LWZ suggests 2 breaks. Therefore, we choose 5 breaks for DM/$. For Yen/$, $\sup F_T(l+1|l)$ is significant when *l* = 3 but not significant when *l* = 4, which suggest 4 breaks. We follow BIC to choose the 5 breaks for Yen/$. For Yen/DM, $\sup F_T(l+1|l)$ suggest 4 breaks as well as BIC. Hence 4 breaks should be chosen for Yen/DM.

In Table 4 we also report the estimates of the break dates with their respective 90% confidence intervals. The break dates estimated for DM/$ and Yen/$ are very similar, which suggests common break dates for the process: May 1989, March – May 1991, March 1993, June – August 1995, and May – July 1997. The estimates of the mean parameters ($\hat{c}_j$) for regimes (*m* + 1) are also provided on the bottom of Table 4. Figure 6 presents the graphs for the logarithmic realized volatility and the estimated $\hat{c}$ value. The mean breaks are coincided with country specific or worldwide economics or financial crisis, i.e. Asian financial crisis occurred in July 1997.

3.3. Long Memory Estimation After Adjusting for Structural Breaks

The sixth column in Table 3 shows the long memory parameter estimates for the three series after adjustment for the estimated structural breaks. The parameter *d* is estimated using the residual series $y_t - \hat{c}_j$. All estimates of *d* are lower than the estimates using non-break adjusted series. Although *d* is not reduced significantly by the Whittle, ARFIMA, and SEMIFAR methods (we will explain this in Section 3.4), *d* has dropped substantially by the GPH method. In addition, the test statistics (DM/$: 1.467, Yen/$: 1.472, and Yen/DM: 1.434) from the rescaled range (R/S) test show that we can not reject the null hypothesis for the absence of long memory. Figure 7 displays the autocorrelation function for the adjusted volatility series. Compared to Figure 5 for the autocorrelation before adjustment for breaks, it is evident that the persistence of volatility has been reduced after removing the estimated breaks.

Furthermore, from Figure 2, there might be an upward trend in the volatility series, especially in Yen/DM series. We use the SEMIFA model with flexible trend by Beran and Ocker (2001) mentioned in Section 2.5 to test this possibility. The results for the estimated trend are shown in Figure 8. We see that the trend is not statistically significant. It is worth noting that Beran and Ocker' (BO) method is an alternative to the BP model. The BP model gives abrupt change whereas the BO model admits a smoother flexible trend. Our results show that the realized volatility series fit the BP model better than the BO model.

3.4. Monte Carlo Simulation for Long Memory Process

We discussed previously that structural change is easily confused with long memory. Granger and Hyung (2004) pointed out that there exists another perplexity: a long memory model without breaks may cause breaks to be detected spuriously by standard estimation methods. To illustrate this phenomenon, we generated six long memory series with d = 0.1, 0.2, 0.3, 0.35, 0.4, 0.45, respectively, with mean: -0.5, standard deviation: 0.4, and sample size: 3,045. These series, which are similar to our sample logarithmic realized volatility, are shown in Figure 9. Table 5 shows results for the structural break tests of BP for the different DGPs. The results suggest a positive relationship between the number of breaks and the value of d as found in Granger and Hyung (2004). This reveals the fact that a long memory/fractionally integrated process itself contains some portion of a permanent shock, which often appears as a break in some situations.[7] The above Monte Carlo evidence shows that long memory provide a good simple alternative of in-sample fit for the true structural-break DGP when we have little knowledge for the past break dates and size.[8]

Next, as mentioned in Section 3.3, we notice that the Whittle, ARFIMA, and SEMIFAR methods gave very different estimates of d than the GPH method in the break adjusted data. We estimate the long memory parameters from the simulated data: ARFIMA (0, 0.45, 0), ARFIMA (1, 0.45, 1) with AR coefficient 0.3 and MA coefficient 0.5, and ARFIMA (1, 0.45, 0) with AR coefficient -0.1. We select the values for coefficients based on the estimation result in Table 3. These DGPs are also graphed on the bottom of Figure 9. From Table 6, we find the estimates, in particular, for ARFIMA (1, 0.45, 1) are distorted using the Whittle, ARFIMA and SEMIFA methods. The above simulation result presents the biased problems of these methods when the

[7] Currently there is no formal test available for multiple structural changes in the I(d) process with unknown number of breaks. It will be interesting for the future research.
[8] This property, which is trivial here, will become much more important when we discuss the long memory and structural breaks for out-of-sample forecasting in Section 5.

DGP includes moving average process and explains the reason for inconsistent estimates of d in Section 3.3.

4. Forecast Evaluation and Simulation

4.1. Forecast evaluation and comparison

Many models have been provided for forecasting asset return volatility and the success of a volatility model lies in its out-of-sample forecasting power. For example, ABDL propose a trivariate VAR-RV-I(d) (fractionally integrated Gaussian vector autoregressive-realized volatility),

$$\Phi(L)(1-L)^d(Y_t-\mu)=\varepsilon_t \tag{13}$$

where Y_t is (3×1) vector of logarithmic realized exchange rate volatilities; μ is unconditional mean and ε_t is a vector white noise process. They fix the value of d for each series at 0.401, which is also close to our long memory estimates in Table 3. They choose the orders of 5 for the lag polynomials in $\Phi(L)$ to being equal to five days, or one week. They compare with the volatility forecasts from several popular models, and they find that their VAR-RV-I(d) model produces superior out-of-sample forecasts.

Here we assess the forecasting performance from our VAR-RV-Break model,

$$\Phi(L)(Y_t^*-\mu)=\varepsilon_t \tag{14}$$

where Y_t^* is the vector of logarithmic realized exchange rate volatilities after mean break adjustment. Although the Bayesian information criteria select a fourth-order VAR, we use a fifth-order model to compare our result to those in ABDL.[9] Forecasts are obtained by estimating rolling models. We estimate initially over the first 2449 observations, December 2, 1986 to

[9] We also evaluate model by VAR(4). The results are similar to VAR(5).

December 1, 1996, and using the in-sample parameter estimates,[10] one-day-ahead forecasts are made for the next day, say day 2450. The process is then rolled forward 1 day, deleting the first observation and adding on the 2450 observation, the model is re-estimated and the second forecast is made for 2451. The rolling method is repeated until 3045, the end of the out-of-sample forecast period. We get 596 one-step-ahead predictions in the out-of-sample period, which is from December 2, 1996 to June 30, 1999. If we assume that the future break dates and sizes are known, we then adjust them back based on the given out-of-sample mean breaks.

In Figure 11, we plot the DM/\$, Yen/\$, and DM/Yen realized volatility along with the corresponding one-day-ahead VAR-RV-Break forecasts. It appears that our forecasts capture movement of the realized volatilities well. Next, to determine which model provides more information about the future value, we use the encompassing regression[11] by Mincer and Zarnowitz (1969),

$$\mathrm{vol}_{t+1,i} = \beta_0 + \beta_1 \mathrm{vol}_{t+1|t,i}^{VAR-RV-Break} + \beta_2 \mathrm{vol}_{t+1|t,i}^{Model} + \varepsilon_t \qquad (15)$$

where we denote our benchmark VAR-RV-Break model prediction of future volatility by $\mathrm{vol}_{t+1|t}^{VAR-RV-Break}$, and future volatility prediction from other candidate methods by $\mathrm{vol}_{t+1|t}^{Model}$. The alternative models are all selected by ABDL and described as follows. First, the VAR-RV-I(*d*) model (13) is the main model proposed by ABDL. Second, the VAR-ABS model is fractionally integrated vector autoregressive using daily absolute returns instead of realized volatility. Third, the GARCH model pioneered by Engle (1982) and Bollerslev (1986) describes short-memory conditional volatility via maximum likelihood procedure as a linear function of past squared forecast errors. Based on 2,449 daily in-sample returns, we get the GARCH (1,1) estimates with

[10] We choose this in-sample period to compare our result to those in ABDL.

[11] This is a regression-based method where the prediction is unbiased only if β_0=0 and β_1=1. When there are more than one forecasting models, additional forecasts are added to the right-hand-side to check for incremental explanatory power. The first forecast is said to subsume information in other forecasts if these additional forecasts do not significantly increase the R^2.

AR polynomial for DM/$, Yen/$, and DM/Yen being 0.986, 0.968, and 0.99, respectively. Fourth, the RiskMetrics model from J. P. Morgan is widely used by practitioners. We get the RiskMetrics daily variances and covariances using exponentially weighted moving averages of the cross products of daily returns by a smoothing factor λ=0.94.[12] Fifth, the fractionally integrated exponential GARCH (FIEGARCH)[13] (1,*d*,0) by Bollerslev and Mikkelsen (1996) is a variant of FIGARCH model by Baillie, Bollerslev, and Mikkelsen (1996). The last one is the high-frequency FIEGARCH model using the "deseasonalized"[14] and "filtered"[15] 30-minutes returns.

For the robustness check, we also present the popular out-of-sample forecast evaluation, relative mean squared error (MSE),

$$\frac{\sum(vol_{t+1} - vol_{t+1|t}^{Model})^2}{\sum(vol_{t+1} - vol_{t+1|t}^{Break})^2} \qquad (16)$$

where the denominator is the benchmark model mean squared forecast error and the numerator is the candidate methods mean squared forecast error. If the relative MSE is less than one, the candidate model forecast is determined to have performed better than the benchmark. The results are presented in Table 7. Our VAR-RV-Break model out-of-sample forecasts perform as well as ABDL's VAR-RV-I(*d*) model, and outperform most of the rest of the models.

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[12] RiskMetrics is a special form of integrated GARCH (IGARCH) in which the intercept is fixed at zero and the coefficient for the squared returns (λ) is 0.94. λ could be interpreted as a persistence parameter. When λ is closer to one, more weight is put on the previous period's estimate relative to the current period's observation, which means it is more persistent.

[13] FIEGARCH has volatility persistence shorter than IGARCH but longer than GARCH. Bollerslev and Mikkelsen (1996) found that FIGARCH outperforms GARCH and IGARCH and FIEGARCH is better than FIGARCH for S&P 500 returns.

[14] The deseasonalization is from the fact that the intraday volatility has obvious "seasonal" components related to the opening and closing hours of exchange worldwide. This intraday patterns damage the estimation of traditional volatility models from the raw high-frequency returns. Following ABDL, we get the seasonal factor by averaging the individual squared returns in the various intra-day intervals. And then we can construct the seasonal adjusted high frequency returns.

[15] To decrease the impact of the serial correlation in high frequency asset returns from different market microstructure frictions, following ABDL, we use simple first order AR "filter" to the high-frequency returns before estimating FIEGARCH model.

First, the regression R^2 from VAR-RV-Break model is similar to that from VAR-RV-I(d) model and is higher than most of the rest models. Second, we can not reject the hypothesis that $\beta_0 = 0$ and $\beta_1 = 1$ in the VAR-RV-Break model using t tests while we reject the hypothesis that $\beta_0 = 0$ and/or $\beta_2 = 1$ for all the other models except the VAR-RV-I(d) model. Third, in the encompassing regression that includes both the break model and an alternative forecast, the estimates for β_1 are closer to unity and the estimates for β_2 are closer to zero. Fourth, including an alternative forecast method has little contribution to increasing R^2. Finally, most of the relative MSEs are bigger than one, which means that VAR-RV-Break model has the smaller MSE than that in other forecasts.

The results in Table 7 show the superior forecasting ability for the VAR-RV-Break model in which the future break dates and sizes are known in the out-of-sample period. This result is consistent with Hyung, Poon and Granger (2006). Without the additional information in detecting out-of-sample breaks, the prediction ability of the VAR-RV-Break would be lessened and its performance depends on the numbers and sizes of the out-of-sample breaks as shown in Table 8. For the DM/$ series, the VAR-RV-Break model still outperforms all the models except the VAR-RV-I(d) model because the out-of-sample break is not large as shown in Figure 6. But for the Yen/$ and Yen/DM, which show larger breaks, the VAR-RV-Break model's prediction ability becomes inferior to the other models (except VAR-ABS). In this case, the VAR-RV-I(d) would be the best forecasting model.

4.2. Forecast simulation for break and long memory models

For the robustness check about the comparison of the VAR-RV-Break and the VAR-RV-I(d) out-of-sample forecasts, we simulate a DGP for an AR(1) process with 3000 observations, AR(1) coefficient: 0.41, unconditional variance: 0.16 and six periods divided by four ad hoc breaks shown in Figure 12.A. Each period's range and mean are as follows: P1[1:700; 0.5], P2[701:1500; -1.3], P3[1501:2000; -0.5], P4[2001:2300; -0.5], P5[2301:2700; -1.2], and P6[2701:3000, 0.7] where P1 to P3 are in-sample period and P4 to P6 are out-of-sample period.

For the AR-Break model, we perform one step ahead forecasts simply based on the true DGP with AR(1) coefficient: 0.41. When the out-of-sample breaks are known, we adjust the mean for the forecast evaluation. For the AR-I(*d*) model, we use in-sample data (Figure 12.A P1 to P3) to estimate the long memory parameter and the AR(1) coefficient. We get d = 0.2697 and AR(1) = 0.2137. We perform one-step-ahead forecasts from the ARFIMA model. Figure 12.B shows the result for period 4 in which the out-of-sample break has not occurred. Whether breaks are known or not, the break model performs a little bid better than I(*d*) model. The relative MSE is 1.02. Surprisingly, in period 5 and 6 after breaks occurred, the I(d) model still accurately predicts while the break model deteriorates substantially when the breaks are unknown.

Note that even though the DGP is pure mean break series without any long memory, we still can get very good out-of-sample forecast performance using simple AR-I(*d*). This result shows that long memory/fractional integrated model will still be the best forecasting model when the true financial volatility series are created by structural breaks and we have little knowledge about break dates and size.

5. Conclusions

In this paper, we explore the existence and of structural changes in realized volatility for the DM/$, Yen/$ and Yen/ DM spot exchange rate realized volatility. First, our analysis has found strong evidence of long memory behavior in exchange rate realized volatility. Second, we test for and estimate a multiple mean breaks model; and we find several common structural breaks within the three series. Third, after adjusting the realized volatility series for the estimated breaks, we find a substantial reduction of persistence in the realized volatility. The evidence suggests that long memory may be caused by the presence of structural breaks. Fourth, the Monte Carlo simulation reports that the long memory model could spuriously produce multiple structure breaks.

Finally, VAR-RV-Break model is superior among most of the current forecasting methods if the future break dates and sizes are known. With little knowledge about break dates and size, the VAR-RV-I(d) model, however, is still the best forecasting model when the true financial volatility series are created by structural breaks.

Table 1. Daily Realized Volatility Distributions

	Mean	S.D.	Skewness	Kurtosis	Q(20)
			Volatility		
DM/$	0.616	0.269	2.111	11.55	6095.6
Yen/$	0.661	0.331	3.323	33.72	6523.2
Yen/DM	0.618	0.279	2.985	32.56	12443.9
			Logarithmic Volatility		
DM/$	-0.562	0.386	0.308	3.49	8627.2
Yen/$	-0.51	0.43	0.217	3.65	9150.1
Yen/DM	-0.565	0.406	0.101	3.38	18402.3

1. The sample is from Dec 1, 1986 to June 30, 1999.
2. The top panel is the distribution of realized standard deviation, $(\text{realized variance})^{1/2}$.
3. The bottom panel is the distribution of logarithmic realized standard deviation.
4. Ljung-Box test statistics for twentieth order serial correlation, Q(20).

Table 2. Realized Volatility Long Memory Parameters before Adjustment

Tests	Series	d	AR(1)	MA(1)
	DM/$	0.3958	N/A	N/A
GPH	Yen/$	0.3812	N/A	N/A
	Yen/DM	0.5426	N/A	N/A
	DM/$	0.3489	N/A	N/A
Whittle	Yen/$	0.3931	N/A	N/A
	Yen/DM	0.4160	N/A	N/A
	DM/$	0.3489	0	0
ARFIMA	Yen/$	0.39	0	0
(p,d,q)	Yen/DM	0.4143	0	0
	DM/$	0.3444	0	0
SEMIFAR	Yen/$	0.3859	0	0
(p,d,0)	Yen/DM	0.4096	0	N/A

1. GPH test is based on Geweke and Porter-Hudak (1983).
2. Whittle's method is based on a frequency domain maximum likelihood estimation of a process i.e. equation (8).
3. ARFIMA model is based on Beran (1995). $\phi(L)(1-L)^{\delta}[(1-L)^{m} y_t - \mu] = \theta(L)\varepsilon_t$ where -0.5 < d < 0.5. The integer m is the number of times that y must be differenced to achieve stationarity, and the long memory parameter is given by $d = \delta + m$. The method uses BIC to choose the short memory parameters p and q. When m = 0, μ is the expectation of y_t ; when m = 1, μ is the slope of linear trend component in y_t .
4. SEMIFAR (Semiparametric Fractional Autoregressive) model is based on Beran and Ocker (2001). $\phi(L)(1-L)^{\delta}[(1-L)^{m} y_t - g(i_t)] = \varepsilon_t$. By using a nonparametric kernel estimate of $g(i_t)$ instead of constant term μ . The method uses BIC to choose the short memory parameter p.

Table 3. Estimations for Long and Short Memory Parameters

		Log Realized Volatility Before Adjustment			Log Realized Volatility After Adjustment			
		d	AR(1)	MA(1)	d	AR(1)	MA(1)	Q(20)
GPH	DM/$	0.4239 (0.0975)	N/A	N/A	0.021 (0.0975)	N/A	N/A	5438
	Yen/$	0.3571 (0.0975)	N/A	N/A	0.0823 (0.0975)	N/A	N/A	5309
	Yen/DM	0.5867 (0.0975)	N/A	N/A	0.0576 (0.0975)	N/A	N/A	5610
Whittle	DM/$	0.3816	N/A	N/A	0.3693	N/A	N/A	
	Yen/$	0.4146	N/A	N/A	0.3975	N/A	N/A	
	Yen/DM	0.4229	N/A	N/A	0.3852	N/A	N/A	
ARFIMA	DM/$	0.3817 (0.0142)	0	0	0.3671 (0.0142)	0	0	
	Yen/$	0.4107 (0.0142)	0	0	0.3945 (0.0142)	0	0	
(*p*,*d*,*q*)	Yen/DM	0.5673 (0.0316)	0.28 (0.05)	0.49 (0.02)	0.3839 (0.0142)	0	0	
SEMIFAR	DM/$	0.3778 (0.0142)	0	N/A	0.367 (0.0142)	0	N/A	
	Yen/$	0.4096 (0.0142)	0	N/A	0.3976 (0.0142)	0	N/A	
(*p*,*d*,0)	Yen/DM	0.4685 (0.0212)	-0.1018 (0.03)	N/A	0.3833 (0.0142)	0	N/A	
R/S Test	DM/$	test stat: 3.4712**	Reject the Null		test stat: 1.4671	Do Not Reject the Null		
	Yen/$	test stat: 2.9046**	Reject the Null		test stat: 1.4718	Do Not Reject the Null		
	Yen/DM	test stat: 4.4041**	Reject the Null		test stat: 1.4337	Do Not Reject the Null		

1. * indicates 5% significance level and ** indicates 1% significance level and the numbers in the parentheses indicate standard errors.
2. GPH, Whittle, ARFIMA, and SEMIFAR models are explained in the detail below the Table 2.
3. *R/S* Test is called rescaled range statistic defined as $R_T = \max_{0 \le j \le T}[\sum_{j=1}^{T}(y_j - j\bar{y})] - \min_{0 \le j \le T}[\sum_{j=1}^{T}(y_j - j\bar{y})]$ and $s_T = [(1/T)\sum(y_t - \bar{y})^2]^{1/2}$ where R is the range, s_T is the sample standard deviation and $\bar{y}$ is the sample mean. We actually use the modified rescaled range statistic $Q_T = R_T / \sigma_T(q)$ where $\sigma_T^2(q) = c_0 + 2\sum_{j=1}^{q} w_j(q)c_j$, c_j is the jth-order sample autocovariance and $w_j(q)$ is the Bartlett window weights.

Table 4. Multiple Structural Changes Test Results

\ Series Statistics	DM/$	Yen/$	Yen/DM
		Tests	
$\sup F_T(1)$	52.58	133.36	305.13
$\sup F_T(2)$	55.64	76.86	246.36
$\sup F_T(3)$	42.37	54.06	188.79
$\sup F_T(4)$	36.92	50.74	149.35
$\sup F_T(5)$	33.94	42.29	110.46
*UD*max	55.64**	133.36**	305.13**
*WD*max	84.95**	133.36**	323.48**
$\sup F_T(2\|1)$	36.23	17.88**	131.56
$\sup F_T(3\|2)$	12.83	7.19	71.95
$\sup F_T(4\|3)$	16.29	26.37	21.31
$\sup F_T(5\|4)$	16.29	8.57	0
		Numbers of Changes Selected	
BIC	**5**	**5**	**4**
LWZ	2	2	2
Sequential	4	2	4
		Multiple Structural Changes Dates Estimation	
$\hat{T}_1$	1989.5.11 [89.2.14-89.11.1]	1989.5.11 [88.12.21-89.9.6]	1989.11.21 [89.11.7-89.12.15]
$\hat{T}_2$	1991.3.18 [90.9.24-91.8.20]	1991.5.16 [91.3.20-91.10.2]	1992.6.10 [92.3.4-92.10.20]
$\hat{T}_3$	1993.3.5 [92.12.7-93.5.14]	1993.3.30 [92.11.25-93.5.21]	1994.5.4 [94.3.24-94.6.1]
$\hat{T}_4$	1995.8.24 [95.6.15-95.11.29]	1995.6.15 [94.8.24-96.6.19]	1997.5.8 [97.4.17-97.5.28]
$\hat{T}_5$	1997.7.10 [97.4.9-89.10.27]	1997.5.7 [97.4.3-97.6.17]	
		Estimations of Mean for Each Regime	
$\hat{c}_1$	-0.662 (0.015)	-0.686 (0.017)	-0.924 (0.012)
$\hat{c}_2$	-0.472 (0.017)	-0.457 (0.018)	-0.5 (0.013)
$\hat{c}_3$	-0.334 (0.016)	-0.687 (0.017)	-0.342 (0.015)
$\hat{c}_4$	-0.553 (0.015)	-0.446 (0.017)	-0.659 (0.012)
$\hat{c}_5$	-0.770 (0.017)	-0.580 (0.018)	-0.205 (0.014)
$\hat{c}_6$	-0.585 (0.017)	-0.205 (0.018)	

1. * indicates 5% significance level
2. ** indicates 1% significance level
3. In bracket are the 90% confidence intervals
4. In parentheses are standard errors
5. Number of Changes Selected From Sequential Method is based on 1% level

Table 5. Estimated Spurious Breaks for Long Memory Simulation

	Breaks exist or not		Number of Breaks Selected			
d	*Ud*max	*Wd*max	$\sup F_T(l+1 \mid l)$	BIC	LWZ	Sequential
0.1	No	No	0	0	0	0
0.2	Yes	Yes	0	2	0	1
0.3	Yes	Yes	3	4	2	3
0.35	Yes	Yes	3	3	3	3
0.4	Yes	Yes	2	4	2	2
0.45	Yes	Yes	3	4	3	3

1. Six different long memory parameters DGP based on Monte Carlo Simulation for 3045 observations.
2. Structural breaks tests are based on Bai and Perron (1998, 2003).
3. The tests are based on 1% significance level.

Table 6. Long Memory Tests for Long Memory Simulation

d=0.45	GPH	Whittle	ARFIMA	SEMIFA	R/S Test
ARMA (0,0)	0.4061 (0.0975)	0.4604	0.4580 (0.0142)	0.4581 (0.0142)	3.3255**
ARMA (1,1)	0.4019 (0.0975)	0.2756	0.3593 (0.0206)	0.3270 (0.021)	2.9011**
AR: 0.3, MA: 0.5			MA: 0.1419 (0.026)	AR: -0.1104 (0.0267)	
ARMA (1,0)	0.6022 (0.0975)	0.4107	0.4698 (0.0210)	0.4613 (0.0211)	3.6268**
AR: -0.1			AR: -0.113 (0.0266)	AR: -0.1060 (0.0268)	

1. Long Memory Test based on long memory DGP with d = 0.45.
2. In parentheses are standard errors.
3. R/S Test results show the test statistics. **meaning significant at 1% level.

Table 7. Out-of-Sample Forecast Evaluation When Future Breaks Are Known

	β_0	β_1	β_2	R^2	Rel MSE
DM/$					
VAR-RV-Break	0.036 (0.048)	0.978 (0.091)	--	0.246	--
VAR-RV-I(d)	0.021 (0.049)	--	0.987 (0.092)	0.249	--
VAR-ABS	0.439 (0.028)	--	0.450 (0.089)	0.028	--
Daily GARCH	0.051 (0.063)	--	0.854 (0.105)	0.096	--
Daily RiskMetrics	0.219 (0.042)	--	0.618 (0.075)	0.097	--
Daily FIEGARCH	0.305 (0.052)	--	0.436 (0.083)	0.037	--
Intraday FIEGARCH deseason/filter	-0.069 (0.060)	--	1.012 (0.099)	0.266	--
VAR-RV-Break + VAR-RV-I(d)	0.021 (0.049)	0.366 (0.332)	0.628 (0.327)	0.250	0.98
VAR-RV-Break + VAR-ABS	0.037 (0.046)	0.980 (0.102)	-0.009 (0.096)	0.246	3.86
VAR-RV-Break + Daily GARCH	-0.041 (0.060)	0.907 (0.120)	0.189 (0.137)	0.249	1.23
VAR-RV-Break + Daily RiskMetrics	-0.004 (0.047)	0.906 (0.119)	0.139 (0.098)	0.250	1.22
VAR-RV-Break + Daily FIEGARCH	0.046 (0.052)	0.987 (0.109)	-0.024 (0.100)	0.246	1.38
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.066 (0.059)	0.369 (0.207)	0.689 (0.217)	0.274	1.08
Yen/$					
VAR-RV-Break	-0.030 (0.106)	1.090 (0.144)	--	0.330	--
VAR-RV-I(d)	-0.006 (0.110)	--	1.085 (0.151)	0.329	--
VAR-ABS	0.349 (0.086)	--	1.256 (0.241)	0.115	--
Daily GARCH	-0.002 (0.147)	--	1.020 (0.187)	0.297	--
Daily RiskMetrics	0.164 (0.108)	--	0.767 (0.131)	0.266	--
Daily FIEGARCH	-0.289 (0.193)	--	1.336 (0.236)	0.373	--
Intraday FIEGARCH deseason/filter	-0.394 (0189)	--	1.647 (0.263)	0.380	--
VAR-RV-Break + VAR-RV-I(d)	-0.024 (0.101)	0.603 (0.564)	0.490 (0.662)	0.331	1.02
VAR-RV-Break + VAR-ABS	-0.058 (0.109)	1.044 (0.148)	0.166 (0.136)	0.331	3.01
VAR-RV-Break + Daily GARCH	-0.103 (0.141)	0.734 (0.131)	0.432 (0.263)	0.348	1.03
VAR-RV-Break + Daily RiskMetrics	-0.048 (0.112)	0.842 (0.102)	0.245 (0.134)	0.340	1.13
VAR-RV-Break + Daily FIEGARCH	-0.279 (0.209)	0.384 (0.260)	0.962 (0.484)	0.385	0.95
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.395 (0.252)	-0.007 (0.375)	1.656 (0.734)	0.380	1.06
DM/Yen					
VAR-RV-Break	-0.047 (0.096)	1.097 (0.132)	--	0.353	--
VAR-RV-I(d)	-0.047 (0.101)	--	1.146(0.143)	0.355	--
VAR-ABS	0.405 (0.062)	--	1.063 (0.175)	0.119	--
Daily GARCH	0.243 (0.092)	--	0.692 (0.119)	0.300	--
Daily RiskMetrics	0.248 (0.084)	--	0.668 (0.107)	0.286	--
Daily FIEGARCH	0.101 (0.105)	--	0.918 (0.144)	0.263	--
Intraday FIEGARCH deseason/filter	-0.231 (0.150)	--	1.455 (0.217)	0.404	--
VAR-RV-Break + VAR-RV-I(d)	-0.054 (0.099)	0.483 (0.452)	0.650 (0.548)	0.357	1.04
VAR-RV-Break + VAR-ABS	-0.044 (0.094)	1.107 (0.148)	-0.028 (0.140)	0.353	3.60
VAR-RV-Break + Daily GARCH	-0.021 (0.082)	0.816 (0.135)	0.235 (0.167)	0.365	1.16
VAR-RV-Break + Daily RiskMetrics	-0.029 (0.089)	0.860 (0.117)	0.199 (0.121)	0.362	1.21
VAR-RV-Break + Daily FIEGARCH	-0.063 (0.106)	0.978 (0.118)	0.141 (0.143)	0.355	1.01
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.228 (0.156)	0.232 (0.294)	1.197 (0.530)	0.407	1.08

1. In parentheses are standard errors.

Table 8. Out-of-8ample Forecast Evaluation When Future Breaks Are Unknown

	β_0	β_1	β_2	R^2	Rel MSE
DM/$					
VAR-RV-Break	0.057 (0.050)	0.879 (0.088)	--	0.214	--
VAR-RV-I(d)	0.021 (0.049)	--	0.987 (0.092)	0.249	--
VAR-ABS	0.439 (0.028)	--	0.450 (0.089)	0.028	--
Daily GARCH	0.051 (0.063)	--	0.854 (0.105)	0.096	--
Daily RiskMetrics	0.219 (0.042)	--	0.618 (0.075)	0.097	--
Daily FIEGARCH	0.305 (0.052)	--	0.436 (0.083)	0.037	--
Intraday FIEGARCH deseason/filter	-0.069 (0.060)	--	1.012 (0.099)	0.266	--
VAR-RV-Break + VAR-RV-I(d)	0.018 (0.050)	0.057 (0.179)	0.933 (0.196)	0.249	0.95
VAR-RV-Break + VAR-ABS	0.065 (0.047)	0.895 (0.103)	-0.056 (0.104)	0.214	3.73
VAR-RV-Break + Daily GARCH	-0.002 (0.060)	0.814 (0.131)	0.160 (0.161)	0.216	1.19
VAR-RV-Break + Daily RiskMetrics	0.029 (0.046)	0.814 (0.131)	0.115 (0.116)	0.216	1.18
VAR-RV-Break + Daily FIEGARCH	0.069 (0.051)	0.890 (0.111)	-0.029 (0.108)	0.214	1.33
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.072 (0.059)	0.140 (0.189)	0.888 (0.210)	0.267	1.05
Yen/$					
VAR-RV-Break	-0.040 (0.127)	1.419 (0.218)	--	0.262	--
VAR-RV-I(d)	-0.006 (0.110)	--	1.085 (0.151)	0.329	--
VAR-ABS	0.349 (0.086)	--	1.256 (0.241)	0.115	--
Daily GARCH	-0.002 (0.147)	--	1.020 (0.187)	0.297	--
Daily RiskMetrics	0.164 (0.108)	--	0.767 (0.131)	0.266	--
Daily FIEGARCH	-0.289 (0.193)	--	1.336 (0.236)	0.373	--
Intraday FIEGARCH deseason/filter	-0.394 (0189)	--	1.647 (0.263)	0.380	--
VAR-RV-Break + VAR-RV-I(d)	-0.000 (0.120)	-0.042 (0.151)	1.111 (0.146)	0.329	0.67
VAR-RV-Break + VAR-ABS	-0.156 (0.137)	1.242 (0.212)	0.577 (0.155)	0.282	1.98
VAR-RV-Break + Daily GARCH	-0.153 (0.147)	0.688 (0.129)	0.686 (0.203)	0.327	0.68
VAR-RV-Break + Daily RiskMetrics	-0.096 (0.132)	0.841 (0.137)	0.471 (0.108)	0.319	0.74
VAR-RV-Break + Daily FIEGARCH	-0.360 (0.185)	0.458 (0.146)	1.084 (0.295)	0.387	0.62
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.399 (0.190)	-0.371 (0.260)	1.961 (0.450)	0.384	0.70
DM/Yen					
VAR-RV-Break	-0.052 (0.138)	1.486 (0.248)	--	0.227	--
VAR-RV-I(d)	-0.047 (0.101)	--	1.146(0.143)	0.355	--
VAR-ABS	0.405 (0.062)	--	1.063 (0.175)	0.119	--
Daily GARCH	0.243 (0.092)	--	0.692 (0.119)	0.300	--
Daily RiskMetrics	0.248 (0.084)	--	0.668 (0.107)	0.286	--
Daily FIEGARCH	0.101 (0.105)	--	0.918 (0.144)	0.263	--
Intraday FIEGARCH deseason/filter	-0.231 (0.150)	--	1.455 (0.217)	0.404	--
VAR-RV-Break + VAR-RV-I(d)	-0.030 (0.131)	-0.087 (0.175)	1.190 (0.096)	0.355	0.56
VAR-RV-Break + VAR-ABS	-0.106 (0.142)	1.259 (0.269)	0.484 (0.143)	0.247	1.93
VAR-RV-Break + Daily GARCH	0.004 (0.123)	0.655 (0.162)	0.519 (0.110)	0.325	0.62
VAR-RV-Break + Daily RiskMetrics	-0.041 (0.132)	0.762 (0.177)	0.485 (0.085)	0.324	0.65
VAR-RV-Break + Daily FIEGARCH	-0.140 (0.148)	0.795 (0.194)	0.631 (0.107)	0.303	0.54
VAR-RV-Break + Intraday FIEGARCH deseason/filter	-0.188 (0.133)	-0.261 (0.192)	1.608 (0.295)	0.407	0.58

1. In parentheses are standard errors.

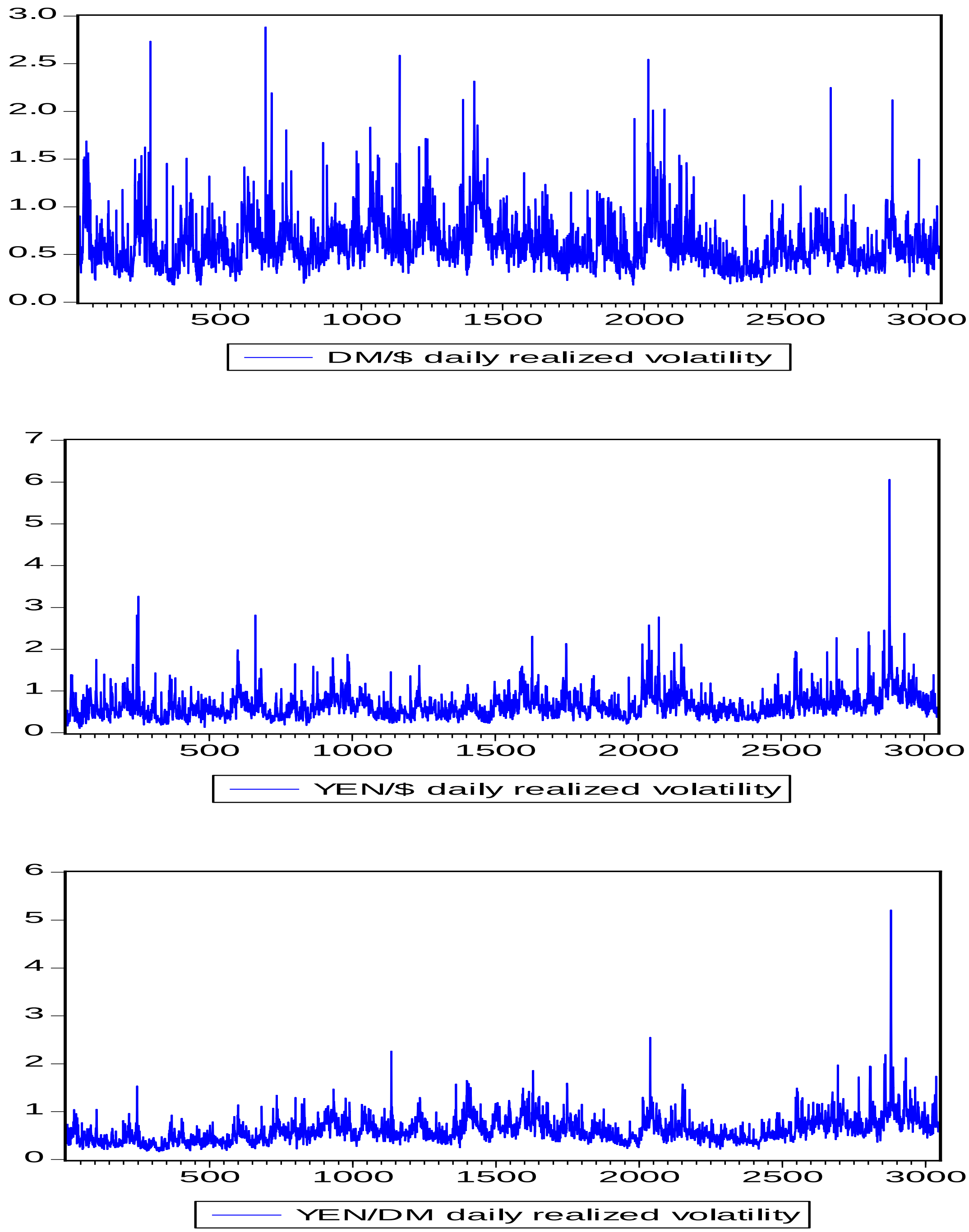


Figure 1. Daily Exchange Rate Realized Volatility
(1986.12.2 – 1999.6.30; 3045 observations)

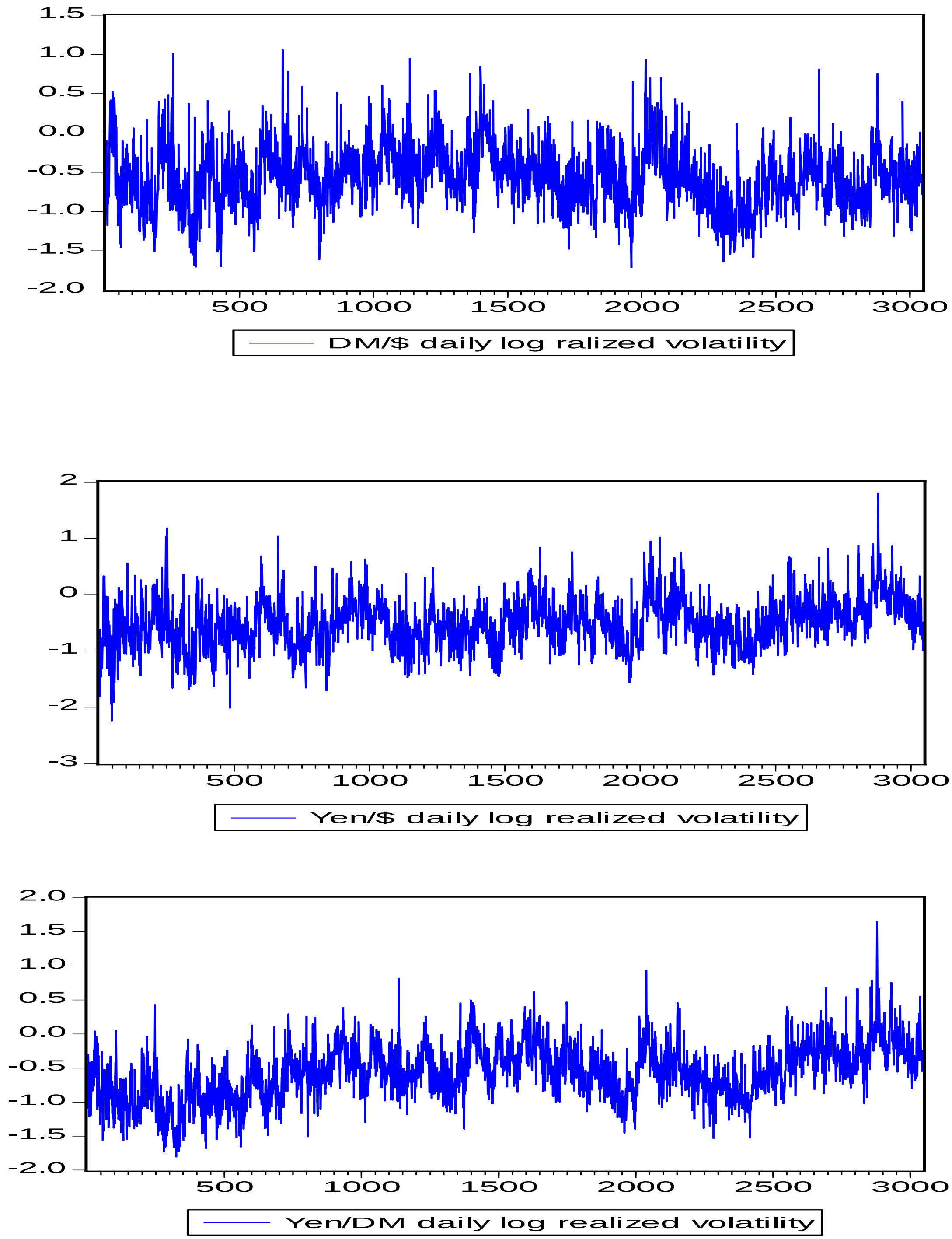


Figure 2. Daily Exchange Rate Log Realized Volatility
(1986.12.2 – 1999.6.30)

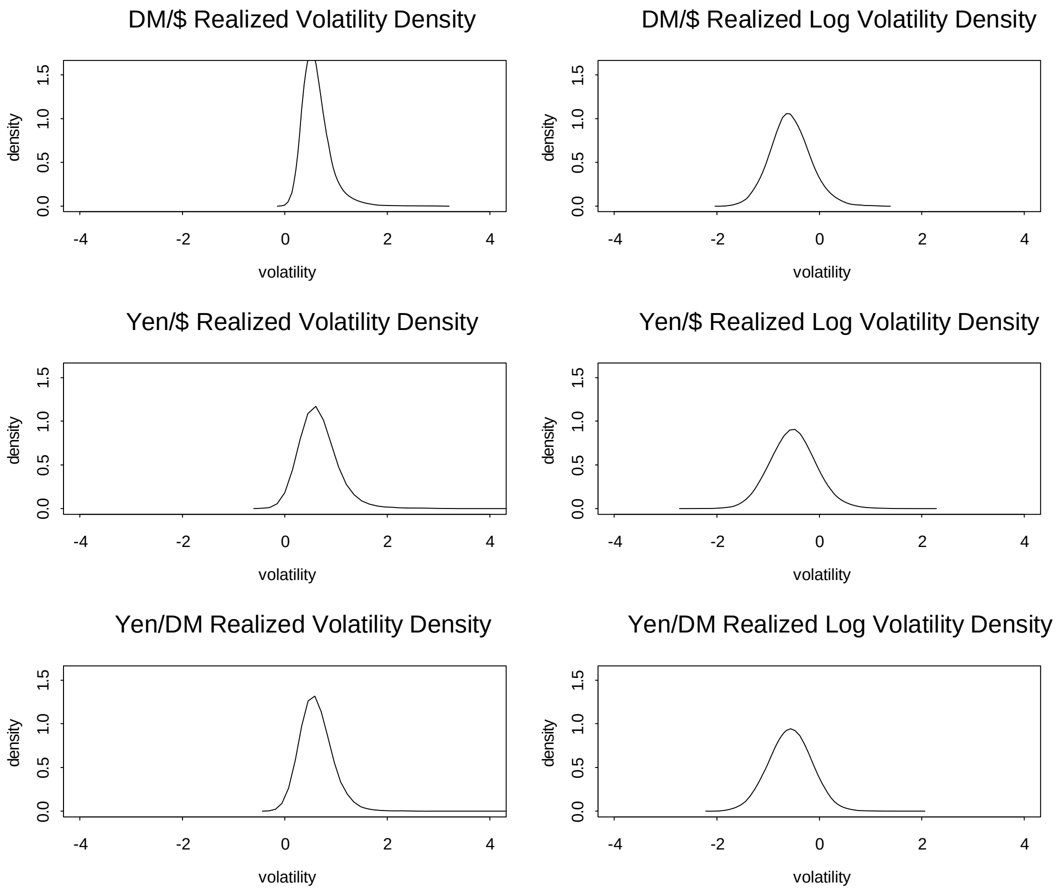


Figure 3. Realized Volatility Distributions

Notes: The figure shows kernel estimates of the density of daily DM/$, Yen/$, and Yen/DM realized volatility. The sample extends from December 2, 1986 to June 30, 1999.

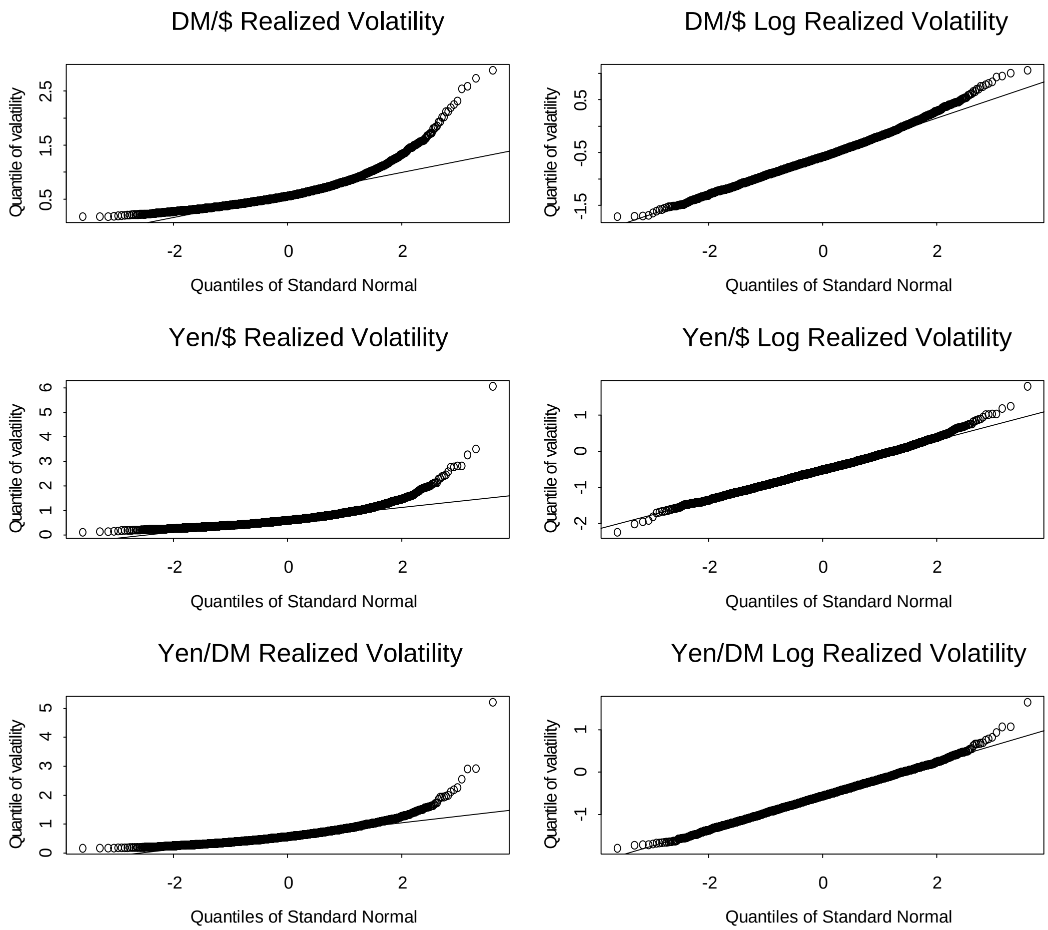


Figure 4. QQ Plot for Realized Volatility

Notes: Quantiles of daily realized volatilities and logarithmic realized volatility from extends from December 2, 1986 to June 30, 1999 against the corresponding quantiles from a standard normal distribution..

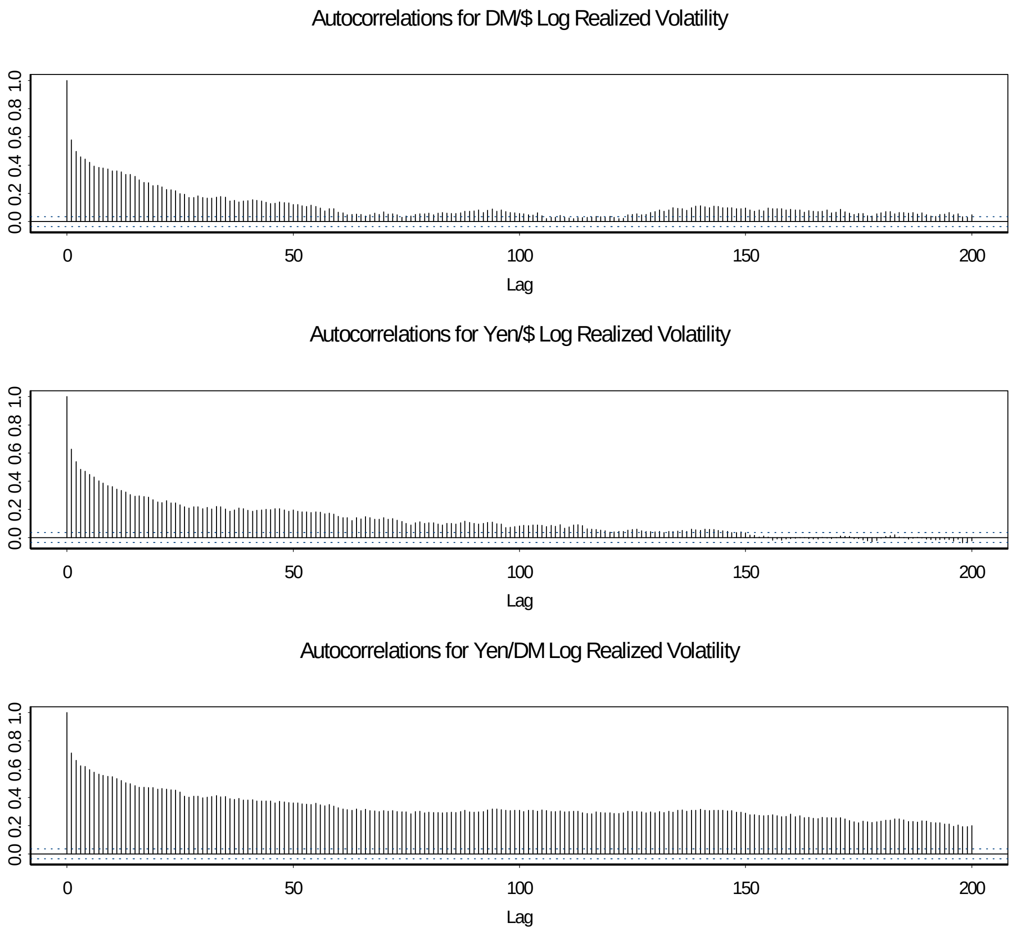


Figure 5. Autocorrelations for Log Realized Volatility

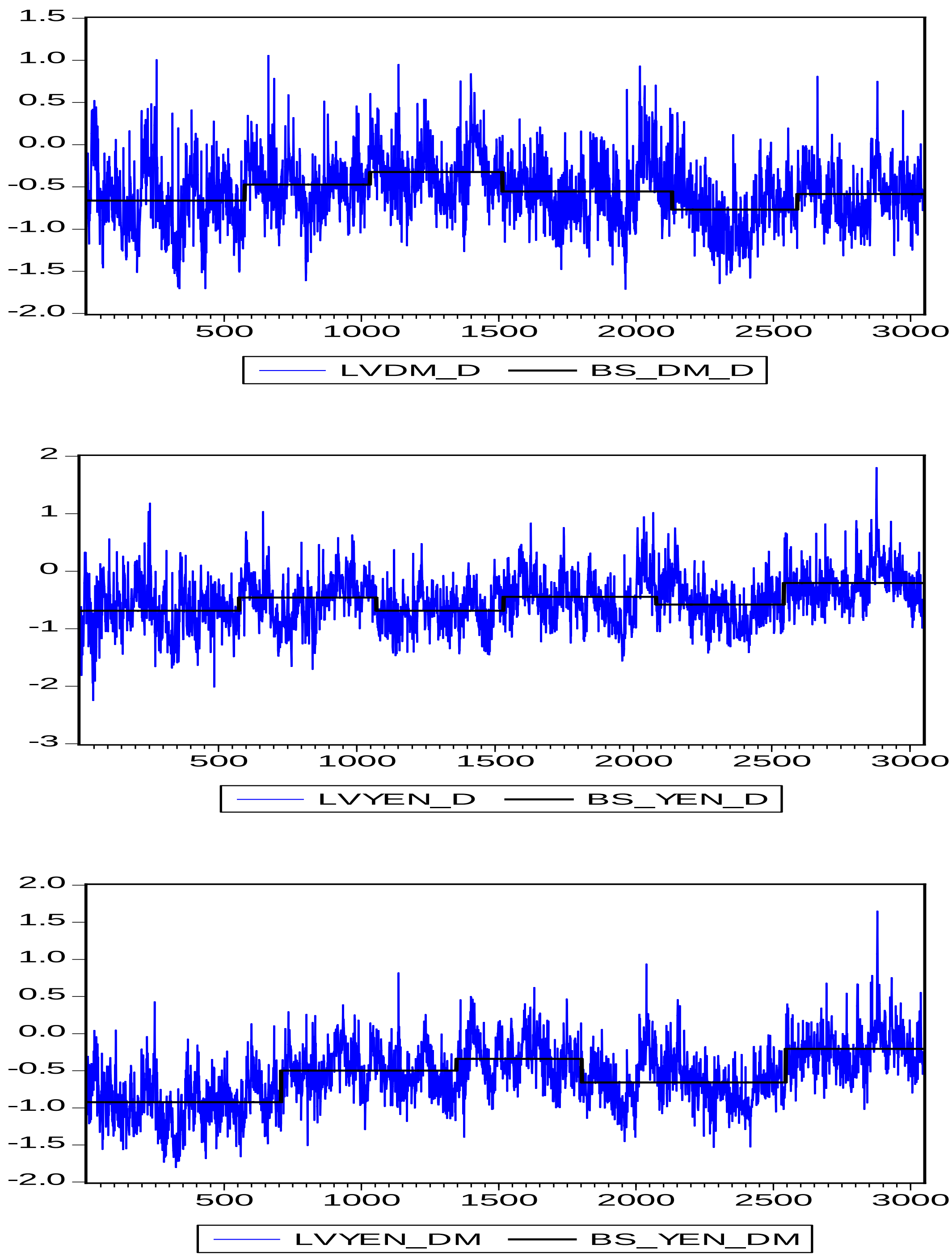


Figure 6. Estimated Structural Breaks Means and Dates for Daily Exchange Rate Log Realized Volatility
(1986.12.2 – 1999.6.30)

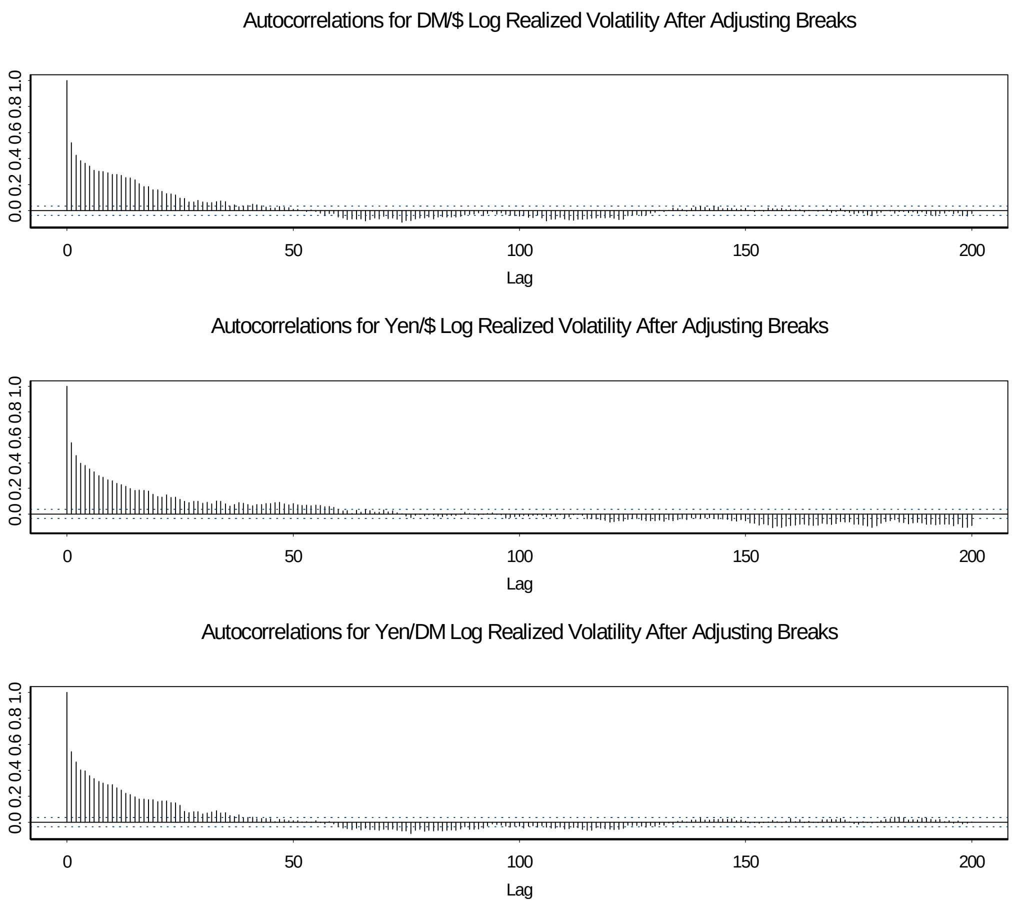


Figure 7. Autocorrelations for Log Realized Volatility After Adjusting for Structural Breaks

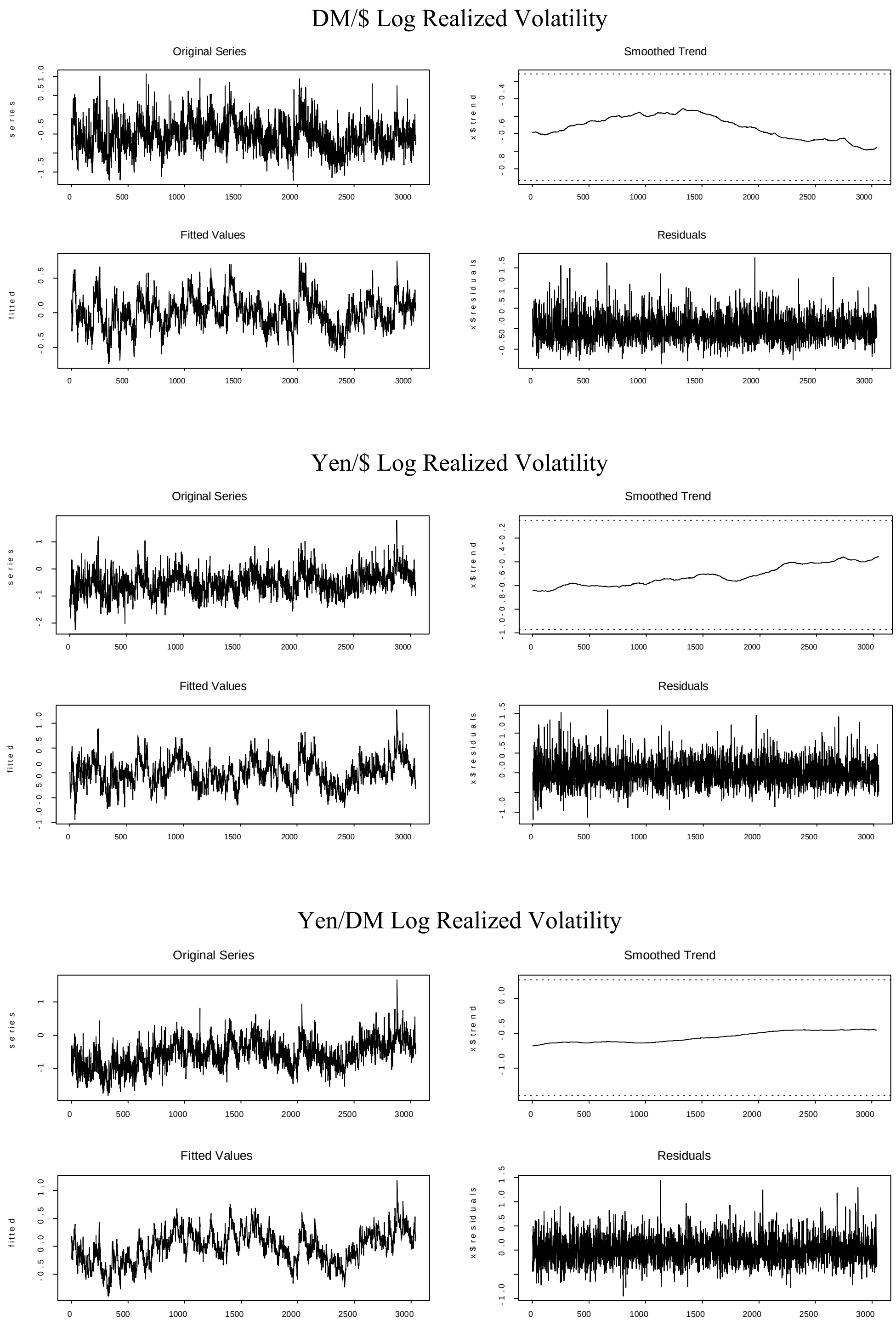


Figure 8. Semiparametric Fractional Autoregressive Model Decomposition

Notes: Based on Beran, Feng and Ocker's method (1998)

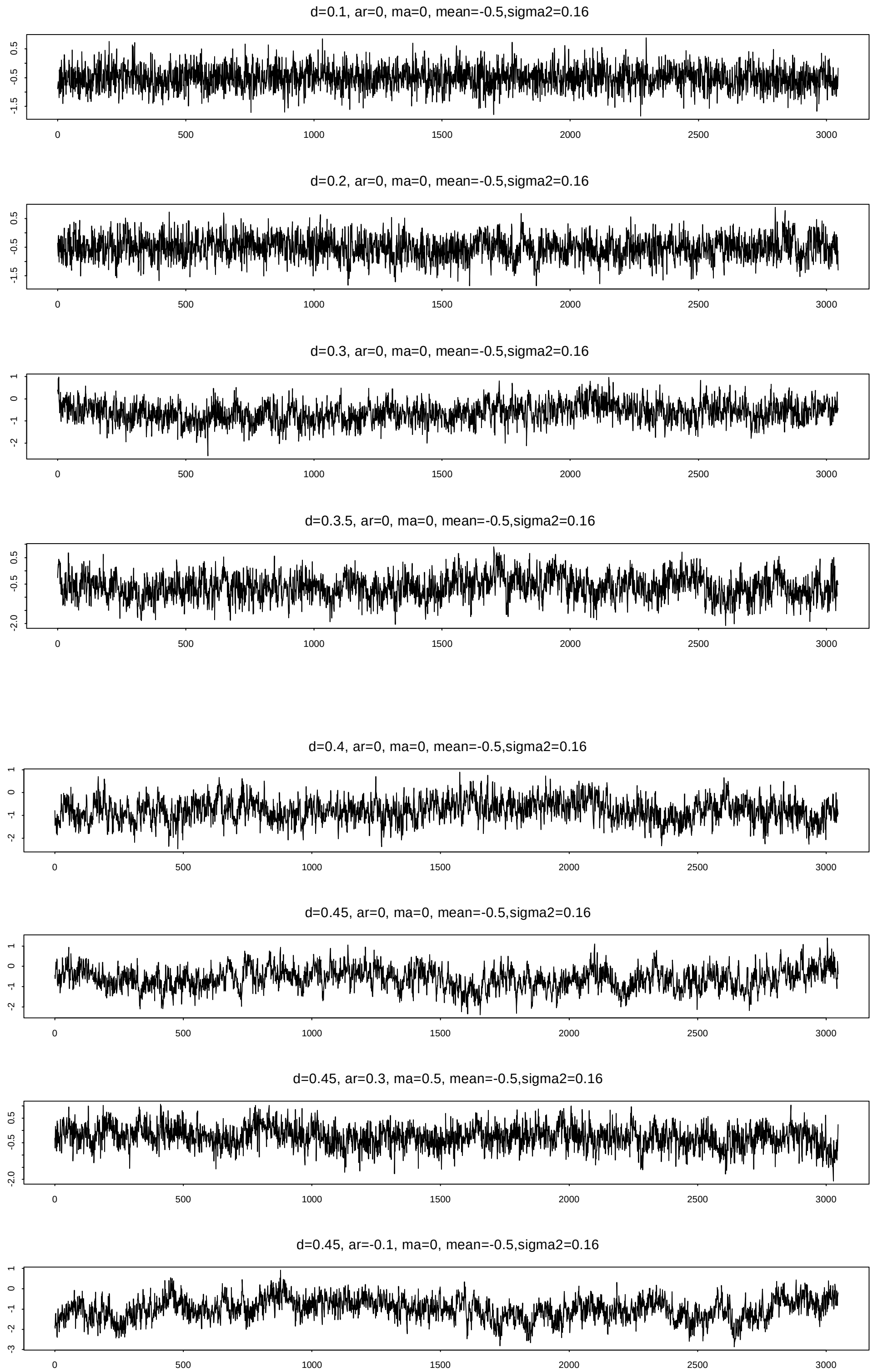


Figure 9. Monte Carlo Simulation for Long Memory Processes

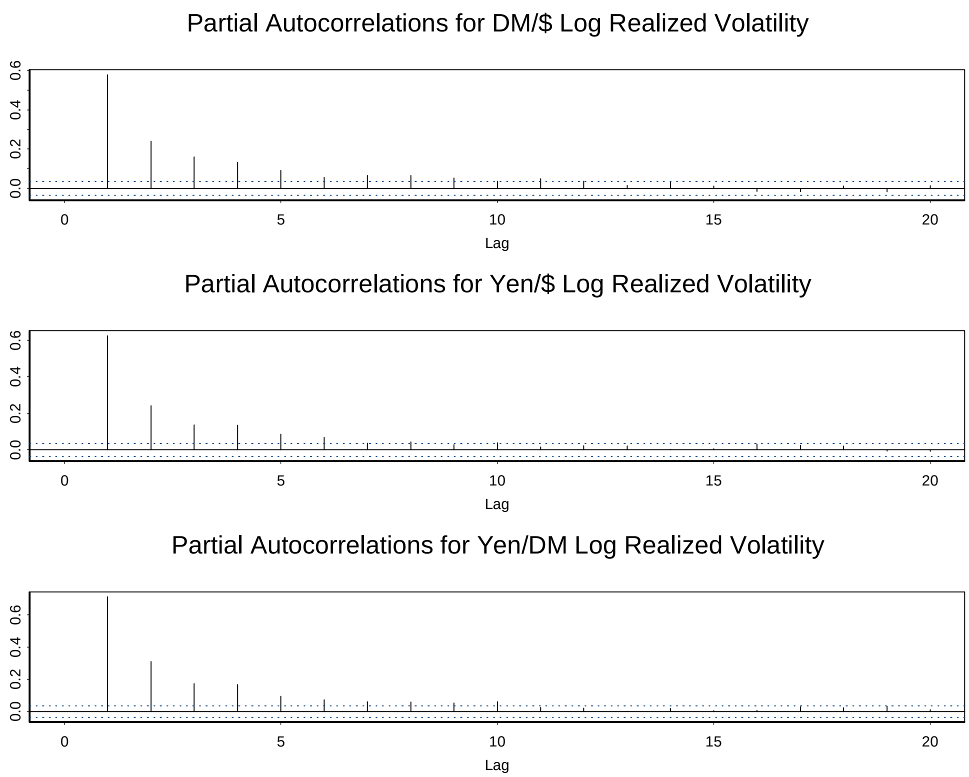


Figure 10. Partial Autocorrelation Function for Log Realized Volatility

DM/$ realized volatility

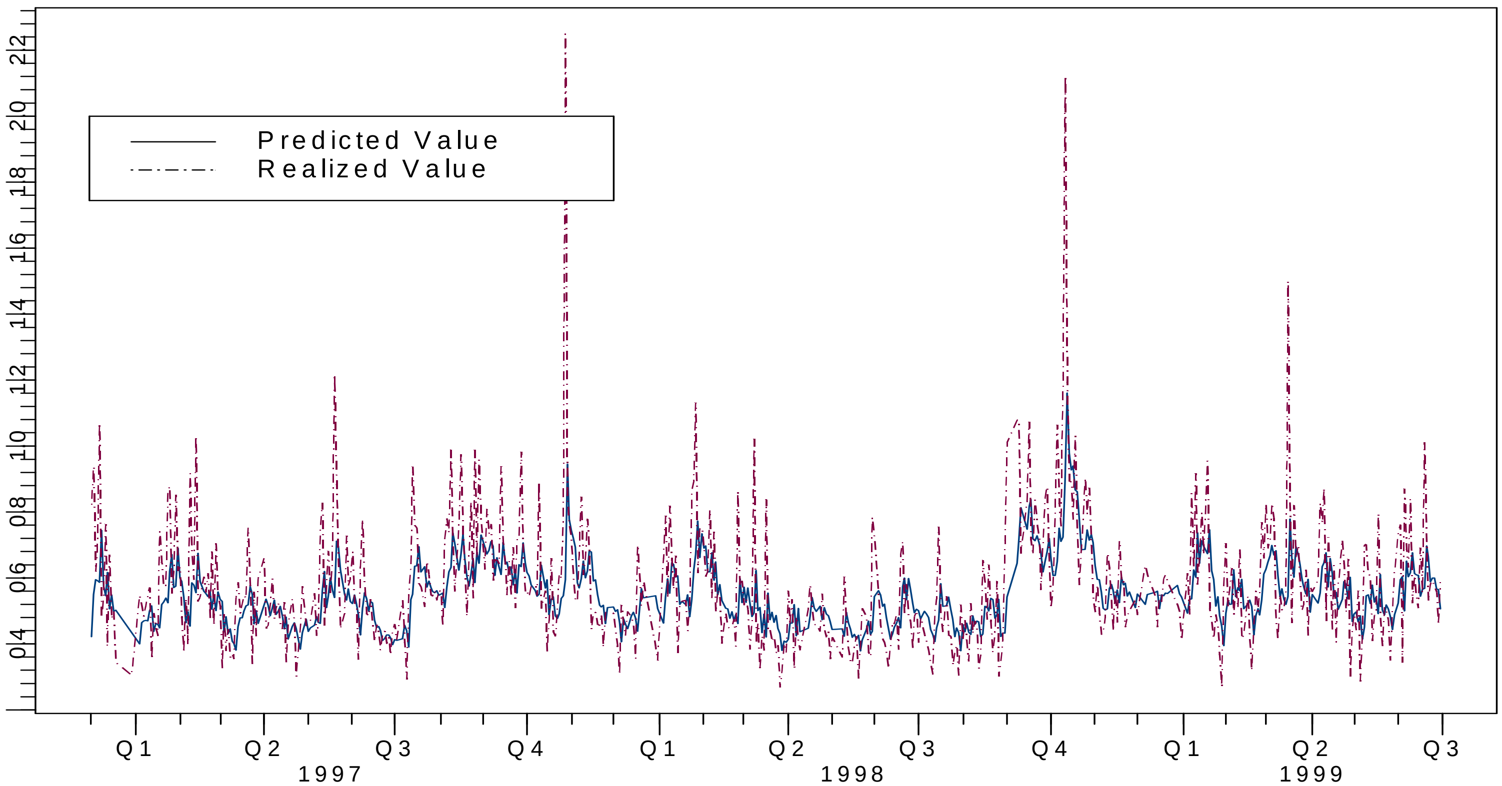


Figure 11.A. Realized Volatility and Out-of-Sample VAR-RV-Break Forecasts

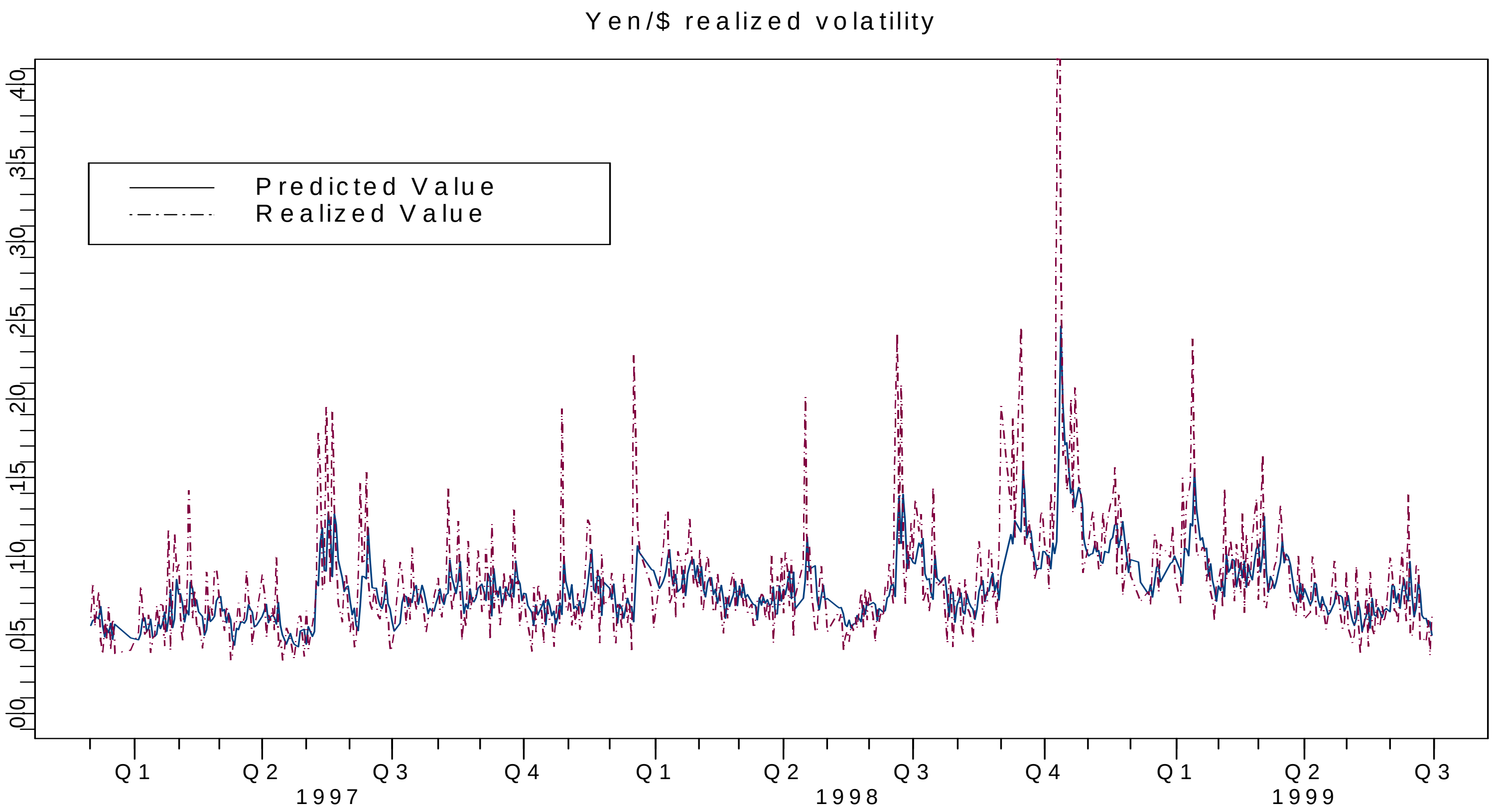


Figure 11.B. (Continued)

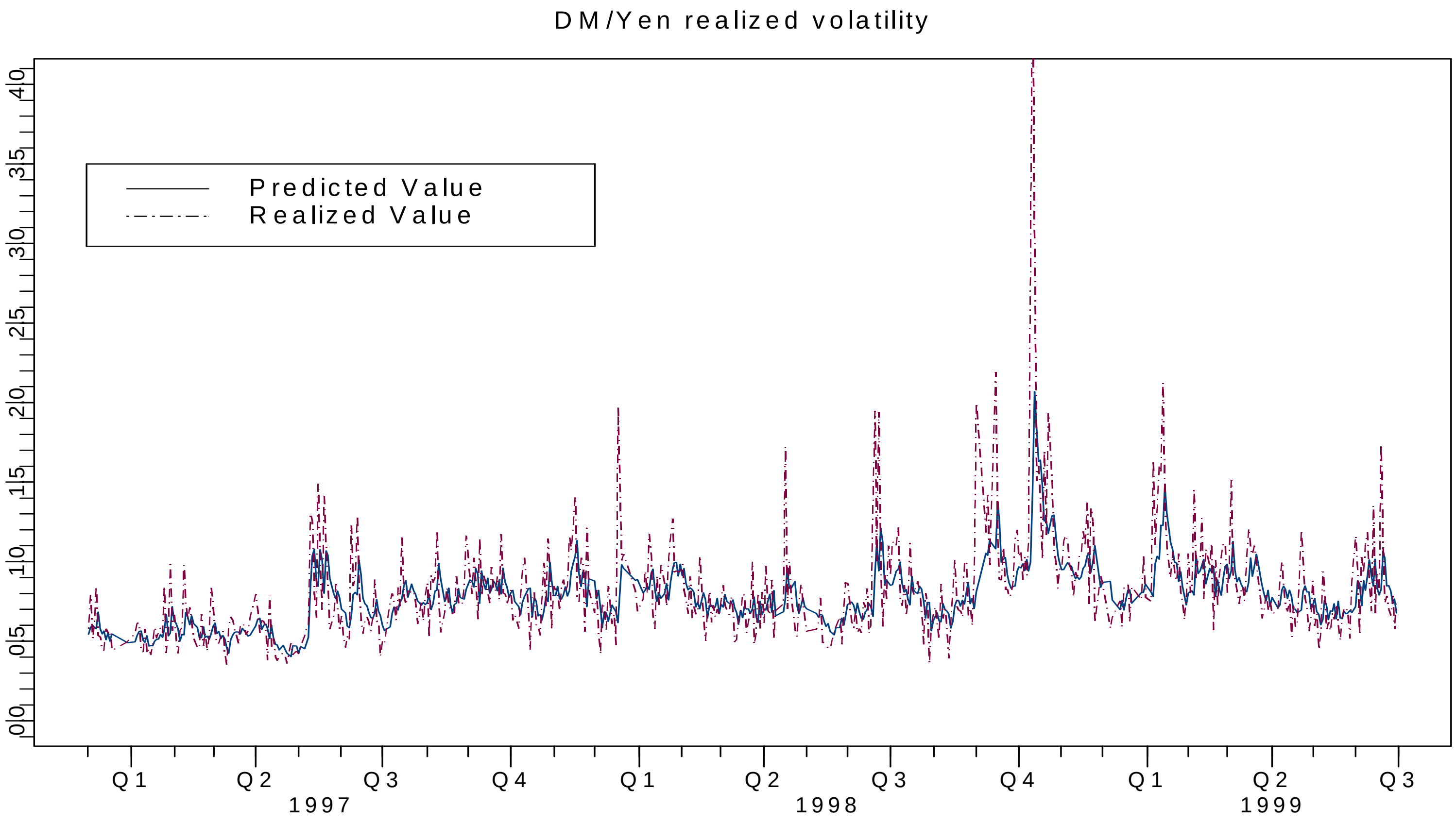
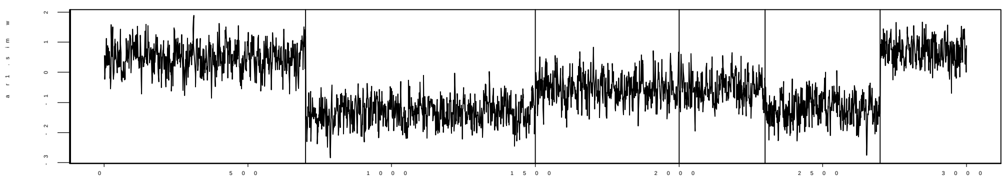
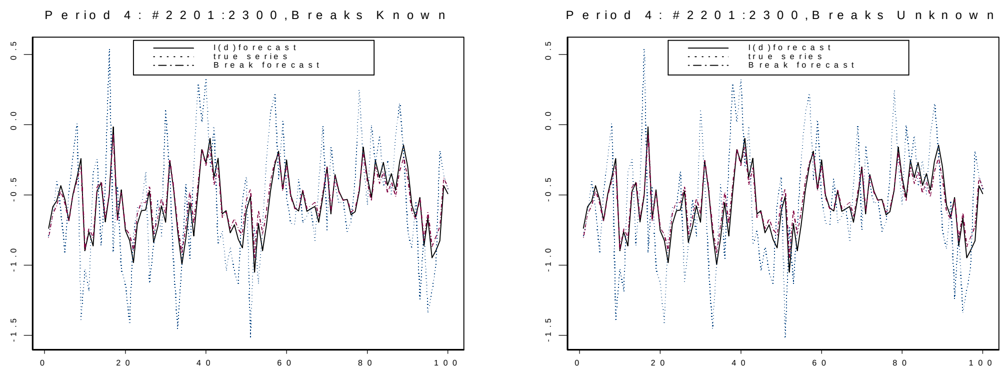


Figure 11.C. (Continued)

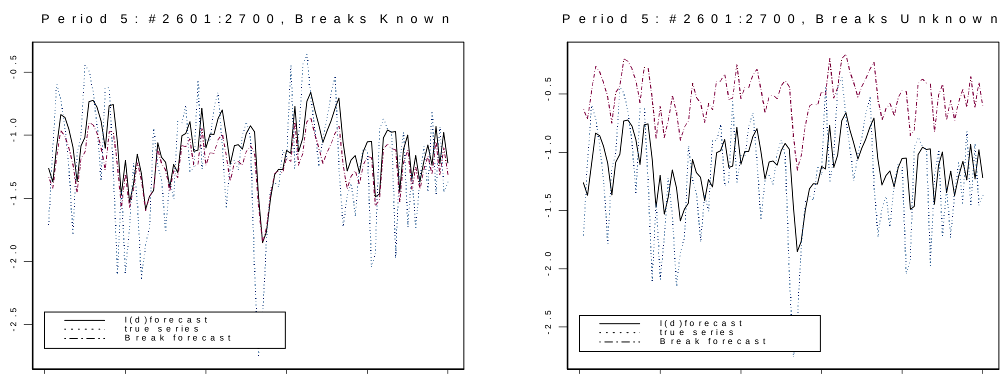
A. Simulated Mean Breaks Series



B.



C.



D.

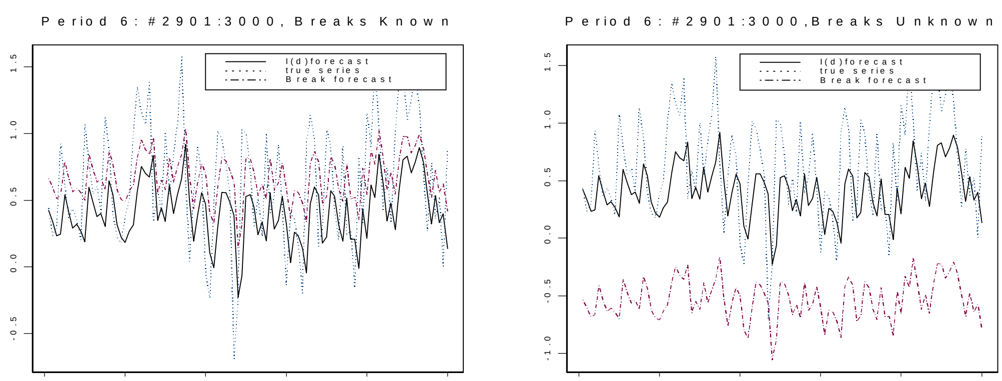


Figure 12. Out of Sample Forecast Evaluation from Simulation

References

Andersen, T. G., and T. Bollerslev. (1997). "Heterogeneous Information Arrivals and Return Volatility Dynamics: Uncovering the Long-Run in High Frequency Returns," *Journal of Finance,* 52:3, pp. 975-1005.

Andersen, T. G., T. Bollerslev, F.X. Diebold, and P. Labys. (2003). "Modeling and Forecasting Realized Volatility," *Econometrica,* 71, pp. 529-626.

Baillie, T.R. (1996). "Long Memory Processes and Fractional Integration in Econometrics," *Journal of Econometrics,* 73, pp. 5-59.

Baillie, T.R., Bollerslev, T., and H.O. Mikkelsen. (1996). "Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics,* 74, pp. 3-30.

Bai, J., and P. Perron. (1998). "Estimating and Testing Linear Models with Multiple Structural Changes," *Econometrica,* 66, pp. 47-78.

Bai, J., and P. Perron. (2003). "Computation and Analysis of Multiple Structural Change Models," *Journal of Applied Econometrics,* 18, pp. 1-22.

Bandi, F. M., and J. R. Russell. (2003). "Microstructure Noise, Realized Volatility, and Optimal Sampling," Working paper.

Barndorff-Nielsen, O. E., and N. Shephard. (2001). "Non-Gaussian Ornstein-Uhlenbeck-Based Models and Some of Their Uses in Financial Economics," *Journal of the Royal Statistical* Society, Series B, 63, pp. 167-241.

Barndorff-Nielsen, O. E., and N. Shephard. (2002). "Econometric Analysis of Realised Volatility and its Use in Estimating Stochastic Volatility Models," *Journal of the Royal Statistical Society, Series B,* 64, part 2, pp. 253-280.

Barndorff-Nielsen, O. E., and N. Shephard. (2004). "Econometric Analysis of Realized Covariation: High Frequency Based Covariance, Regression, and Correlation in Financial Economics," *Econometrica,* 72:3, pp. 885-925.

Beran, J. (1995). "Maximum Likelihood Estimation of the Differencing Parameter for Invertible Short and Long Memory Autoregressive Integrated Moving Average Models," *Journal of Royal Statistical Society Series B*, 57:4, pp.659-72.

Beran, J., and D. Ocker. (2001). "Volatility of Stock-Market Indexes—An Analysis Base on SEMIFAR Models," *Journal of Business and Economic Statistics*, 19:1, pp 103-116.

Bollerslev, T. (1986). "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics,* 31, pp. 307-27.

Bollerslev, T., and H. O. Mikkelsen. (1996). "Modeling and Pricing Long Memory in Stock Market Volatility," *Journal of Econometrics,* 73, pp. 151-84.

Choi, K., and E. Zivot. (2006). "Long Memory and Structural Changes in the Forward Discount: An Empirical Investigation," *Journal of International Money and Finance,* forthcoming.

Diebold, F. X., and A. Inoue. (2001). "Long Memory and Regime Switching," *Journal of Econometrics,* 105:1, pp. 131-59.

Ding, Z., C. W. J. Granger, and R. F. Engle. (1993). "A Long Memory Property of Stock Market Returns and a New Model," *Journal of Empirical Finance,* 1, pp. 83-106.

Engle, R.F. (1982). "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica*, 50:4, pp. 987-1007.

Geweke, J., and S. Porter-Hudak. (1983). "The estimation and application of long memory time series Models," *Journal of Time Series Analysis,* 4, pp 221-237.

Granger, C.W.J., and R. Joyeux. (1980). "An introduction to long memory time series models and fractional differencing," *Journal of Time Series Analysis,* 1, pp 15-39.

Granger, C.W.J., and Z. Ding. (1996). "Varieties of Long Memory Models," *Journal of Econometrics,* 73, pp. 61-77.

Granger, C.W.J., and N. Hyung. (2004). "Occasional Structural Breaks and Long Memory with an Application to the S&P 500 Absolute Stock Returns," *Journal of Empirical Finance,* 11, pp. 399-421.

Hosking, J. R. M. (1981). "Fractional Differencing," *Biometrika*, 68, pp. 165-76.

Hyung, N., S. H. Poon, and C. W. J. Granger. (2006). "A Source of Long Memory in Volatility," Working paper.

Mincer, J., and V. Zarnowitz. (1969). "The Evaluation of Economic Forecasts," in *Economic Forecasts and Expectations,* edited by W. F. Sharpe, and C. M. Cootner. Englewood Cliffs, New Jersey: Prentice-Hall.

Perron, P. (1989). "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis," *Econometrica,* 57, pp. 1361-1401.

Robinson, P. M. (1995). "Log-Periodogram Regression of Time Series with Long-Range Dependence," *Annals of Statistics,* 23, pp. 1048-72.

Zivot, E., and D. W. K. Andrews. (1992). "Further Evidence on the Great Crash, the Oil Price Shock, and the Unit Root Hypothesis," *Journal of Business and Economic Statistic*, 10:3, pp. 251-70.