Investigating the Effects of Evidence-Based Strategies on Word Problems with Regrouping for Elementary Students with Learning Disabilities

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INVESTIGATING THE EFFECTS OF EVIDENCE-BASED STRATEGIES ON WORD PROBLEMS WITH REGROUPING FOR ELEMENTARY STUDENTS WITH LEARNING DISABILITIES

by

Margaret A. Vanderwarn

B. S., University of Wisconsin, Madison, 1985

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INVESTIGATING THE EFFECTS OF EVIDENCE-BASED STRATEGIES ON WORD PROBLEMS WITH REGROUPING FOR ELEMENTARY STUDENTS WITH LEARNING DISABILITIES

Margaret A. Vanderwarn

Recent emphasis on common core state standards and for solving real-life word problems in math has left teachers searching for effective and efficient ways to approach the challenge of word problem solving. The quest for sound and successful strategies holds especially true in special education. The purpose of this study was to investigate the effects of a combined strategy instruction approach on strengthening problem solving competence of students with math difficulties (MD). Seven students received 14 lessons of explicit instruction embedded with cognitive strategies and paired with a graduated lesson sequence. Four different types of word problem situations involving either addition or subtraction with regrouping were the focus of this study. The independent variable consisted of math instruction in a multiple-baseline design with two replications. Ongoing probes as well as pre- and posttests were administered to evaluate treatment outcomes. Both word problem solving and computation skills were analyzed. All participants improved word problem solving from baseline to intervention yielding a range across participants in mean percentage point increase from baseline to intervention of 15.9 to 82.2. On pretest to posttest for this skill, the percentage point increase ranged from 10 to 74 with a mean increase of 41.4. Most students showed improvement from pretest to posttest for untimed computation skills ability with results ranging from a ten percentage point decline to a 55 percentage point increase and a mean of 20.7 increase. Additionally, all students improved in addition and subtraction with regrouping computation fluency from pretest to posttest revealing a range in percent increase of correct digits per minute from 15% to 750% with a mean increase of 121.8%. All students disclosed high satisfaction with the intervention and with the level of learning incurred. Combining effective evidence-based strategy instruction with a graduated lesson sequence showed promising results for students with MD for solving word problems. Given the small
sample size of this study, more research is needed to substantiate these findings using a larger participation pool.

Month  Year  Approved by Research Committee:

Jerry Wellik  Chairperson
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Your gentle heart is a gift to us all.

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I love each and every one of you for allowing me to share with you along the way.
Learning with you
Is a process of
Putting into words
What we sensed
But did not know
Until you have
Connected the dots
And now we
See the whole

by
Jerry Wellik
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Chapter I

NATURE OF THE PROBLEM

Introduction

In a document written for the honors society’s Pi Lambda Theta publication, *Educational Horizons*, Temple Grandin shared: “Word problems in math were very difficult for me” (Grandin, 2006, p. 330). Grandin grew up with special education needs that were left unmet. It was not until a teacher in high school took interest in her abilities, not her disabilities, that Grandin was redirected on a lifelong path of scientific success and contributions—a story that may have been left untold if not for that teacher. Temple Grandin has since become an enthusiastic spokesperson for people with disabilities. One of the more pronounced tenets of Grandin’s message is her emphasis on how people learn differently, especially those with disabilities. Being taught tools to help students with disabilities succeed is critical to their success and sense of personal worthiness. The earlier students are taught these skills, the less chance there is of them falling so far behind their peers that catching up to them seems an impossibility. The brilliance of Grandin’s scientific discoveries is unquestioned. It is unfortunate however, that a person with such a mind was “teased and miserable” as a child (Grandin, 2006, p. 229) because of her learning struggles.
Similar to Grandin, word problem-solving is difficult for many with learning disabilities. Even with good computational skills, word problems are often seen as a “bottleneck for learning” (Swanson, 2014, p. 832) for students with disabilities. Therefore, this thesis explores concepts and strategies leading to the success of word problem computations for elementary students with disabilities. Such success in the classroom may reduce the level of teasing students with disabilities endure in their school day.

History of the use and importance of mathematical word problems dates back to Babylonia, 1600 BC (Melville, 2006). The significance of students achieving mastery in word problem computation in modern day academics has increased considerably in recent decades. However, results of international comparisons in mathematics show students in the United States ranked at or near the bottom of these lists (Xin & Jitendra, 1999). As a result, a focus on employing evidence-based instructional strategies in core curricular areas, such as mathematics, has been stipulated in the No Child Left Behind Act (NCLB, 2001). However, there still is growing concern in our country over poor achievement in student mathematics skills (National Mathematics Advisory Panel, 2008).

These concerns led to the development of the Common Core State Standards (CCSS) for K-12 instruction in the areas of mathematics and English language arts/literacy (Common Core State Standards Initiative, 2014a). As of 2014, 45 states, the District of Columbia, the Department of Defense Education Activity, and four U.S. territories (American Samoan Islands, Northern Mariana Islands, Puerto Rico, and
U.S. Virgin Islands) have adopted the CCSS for both mathematics and English language arts (Common Core State Standards Initiative, 2014b).

However, CCSS (2014c) offers just a little over a page of text in referencing exactly how the newly developed tome of academic standards applies to the 6.4 million students (as of 2012) with disabilities in the U.S. (Institute of Educational Sciences, 2014). While emphasizing access to rigorous and challenging evidence-based instruction for students with disabilities, the CCSS stipulates these students must also have access to supports and accommodations to fit their individual needs, while promoting high expectations to successfully participate in the newly defined academic expectations. However, students with disabilities, specifically those with math difficulties, perform well behind the grade level of their peers, and fall below the target standards on state tests in math (Fontana, 2005).

In addition, the CCSS has placed particular importance on solving word problems by specifying the instruction of this skill across all grade levels (Common Core State Standards Initiative, 2014d). For example, this skill and other math skills are rigorously tested as a part of the state high school graduation requirement tests in Minnesota (Minnesota Department of Education, 2014). Further significance of these standards has been highlighted by the 2013 public declaration of support for the CCSS for Mathematics by the National Council of Teachers of Mathematics as outlined in its Position Statement *Supporting the Common Core State Standards for Mathematics* (National Council of Teachers of Mathematics, 2014).
Beyond these standards and expectations, solving word problems in real-world contexts can be very important to future adults. Starting out on their own, new high school graduates face more highly technical job markets and make a kaleidoscope of choices for their own personal finances, not to mention the mathematical decisions they will make on a daily basis (i.e., gas mileage, budgeting, hobbies). It is within this context recent research for teaching word problems to students with math difficulties has been explored. Specifically, this introduction will consider the following with regard to instruction of word problems for students with disabilities: (a) definition of word problems, (b) commonly used types of word problems, (c) explicit instruction, (d) cognitive strategy instruction, (e) working memory considerations, and (f) linguistics and reading comprehension considerations.

To begin, Fuchs et al. (2006) defined mathematical word problems as math problems presented in a linguistic fashion in which arithmetic is employed to solve them. In this thesis, I recognize that the CCSS for mathematics has placed an emphasis on mastering real-life word problems across mathematical operations (Common Core State Standards Initiative, 2014a). Yet, many students with math difficulties (MD) struggle with solving word problems (Ferreira, 2009; Miller & Kaffar, 2011a; Miller & Kaffar, 2011b; Miller & Mercer 1993a; Miller & Mercer, 1993b; Swanson, Moran, Bocian, Lussier, & Zheng, 2012).

Griffin and Jitendra (2008) pointed out students with disabilities show weakness in math reasoning—a significant skill used in approaching word problem computation (Swanson, Jerman, & Zheng, 2008; Vilenius-Tuohimaa, Aunola, &
Nurmi, 2008). Furthermore, math reasoning is also cited as a predominant concern throughout the CCSS (2014a) and publications of the NCTM (2014). Therefore, it is important when teaching word problems to students with disabilities to maintain rigor and fidelity regarding these newly adopted expectations by building math reasoning through evidence-based mathematics instruction.

It is important to recognize there are different definitions of word problem types used for instruction. Carpenter and Moser (1984) identified three main types of word problems: (1) Change, (2) Difference, and (3) Combine. These are the same types of word problem categories recommended by the authors of a 2009 practice guide supported by the Institute of Education Sciences (IES) and the U.S. Department of Education (Gersten et al., 2009). However, others have identified four word problem structures used explicitly for addition and subtraction situations: (1) Add to, (2) Part-Part-Whole (Put Together/Take Apart), (3) Take Away (Take From), and (4) Compare (National Math Alliance, 2010). These last four classifications align with the CCSS (2014a), which are: (1) Add To; (2) Take From; (3) Put Together/Take Apart; and (4) Compare.

Solving word problems involves a complicated process comprised of four steps: 1) understanding and representing the nature of the problem, 2) devising a method for solving the problem, 3) taking action and carrying out the plan, and 4) drawing meaning from the solution by accurately interpreting it, which may involve drawing on previous knowledge (Desoete, Roeyers, & de Clercq, 2003). To
successfully address these steps, research has shown the importance of considering certain strategies and approaches relative to the success of this process.

Two instructional models have been researched in relation to solving mathematics problems: Explicit instruction and cognitive strategy instruction. Explicit instruction is defined by the National Mathematics Advisory Panel (2008) as providing: (a) demonstrations and models for solving problems, (b) numerous examples for solving problems, and (c) multiple practice opportunities. In addition, the panel notes the importance of student dialogue of processes they used for solving the problem and includes providing plenty of teacher feedback.

Given the broad nature of explicit instruction and its ubiquitous application for improved instruction of students with disabilities, the usefulness of explicit instruction has been applied to many types of math problem instruction (see the meta-analysis of Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2009; Miller & Hudson, 2006). Moreover, this meta-analysis makes clear the effectiveness of explicit instruction on students with learning disabilities. In a 2012 extension of this synthesis, Zheng, Flynn, and Swanson showed explicit instruction to produce larger effect sizes in outcomes on word problem solving for students with math disabilities.

Cognitive strategy instruction, on the other hand, focuses on the process involved for solving mathematics problems. Models include: read; paraphrase; visualize; hypothesize; estimate; compute; and check (Montague, 1997). Others are: advance organizers; skill modeling; explicit practice; task difficulty control;
elaboration; task reduction; questioning; and providing strategy cues (Gersten et al., 2009).

Of the cognitive strategies emerging in modern-day learning theory, the successful approaches directly tap into the learning process as it relates to short-term (ST), working (W), and long-term (LT) memory (M) (Goodwin, 2014). Information first approaches STM and the success of passing its filter to WM greatly depends upon the emotion associated with it. For example, building a positive rapport with students can enhance this activity. As information advances to WM, success largely depends upon the nonlinguistics associated with it. Furthermore, Goodwin went on to stipulate storing information into LTM can depend upon the personal meaning that information has for the student and their experiential background pertaining to it. However, far and away the most successful route to LTM storage is by using repeated practice. One of the key techniques underpinning the success of explicit instruction in special education is multiple opportunities for practice.

Using Baddeley’s (1986) popular model of the role WM plays in math problem-solving (as cited in Zheng, Swanson, & Marcoulides, 2011), Zheng et al. (2011) demonstrated the three major parts of working memory—executive, phonological loop, and visual-spatial sketchpad—largely influence the success elementary students have while solving mathematical word problems.

However, the particular route information uses to reach LTM is also important to understanding when to choose the most effective strategy for improving student outcomes on learning. This can be a complicated process (Goodwin, 2014). Still,
WM capacity (WMC) can moderate how well cognitive strategies work for individual students with MD (Swanson, 2014). Swanson explained that working memory capacity is drawn upon to a “considerable degree” when solving word problems, and therefore, can slow down this process if WMC is limited.

Although a systematic approach to explicit instruction has been developed (Archer & Hughes, 2011; Archer & Isaacson, 1990), Archer and others stressed the importance of weaving complementary cognitive strategy instruction throughout explicit instruction lessons (Belleza, 1981; Boonen, van Wesel, Jolles, & van der Schoot, 2014; Bryant, Hartman, & Kim, 2003; Carmack, 2011; Ferreira, 2009; Flores, Hinton, & Strozier, 2014; Fuchs et al., 2003; Gersten et al., 2009; Mancl, 2011; Miller & Kaffar, 2011a; Xin & Jitendra, 1999).

To date, a variety of cognitive strategies have been developed and coupled with explicit instruction. First, Miller and Kaffar (2011a) and Carmack (2011) revealed increased competence of addition with regrouping skills for elementary students with math difficulties by incorporating mnemonics. Further success using mnemonics has been demonstrated for teaching subtraction with regrouping to students with learning difficulties (Ferreira, 2009; Mancl, Miller, & Kennedy, 2012; Miller, & Kaffar, 2011b), and for teaching subtraction and multiplication with regrouping (Flores et al., 2014).

This has been shown to aid in successful mastery of word problem instruction as well (Carmack, 2011; Swanson, 2014). For example, use of mnemonics was shown to improve performance in solving word problems for students with learning
difficulties (Miller & Mercer, 1993a). In particular, the use of mnemonics in the form of acronyms improves problem-solving skills (procedural skills). For instance, these authors explained the use of FAST DRAW, a mnemonic successfully used with word problems involving multiplication, in the Strategic Math Series Find what you’re solving for.

1. Find what you’re solving for.
2. Ask yourself, “What are the parts of the problem?”
3. Set up the numbers.
4. Tie down the sign.
5. Discover the sign.
6. Read the problem.
7. Answer, or draw and check.
8. Write the answer.
(Mercer & Miller, 1992, p. 130).

A second cognitive strategy, schema-based instruction (SBI) with visuals, was studied for students with learning disabilities in math when solving word problems. SBI focuses on the common underlying structures in word problems that allow them to be solved according to which category, or schema, they belong (Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007a). Transferring the learned schema into a corresponding visual schematic can be beneficial to learning word problems. There have been studies that produced convincing evidence for this benefit on mathematical instruction (Foster, 2007; Fuchs et al., 2003; Jitendra et al., 2007a; Jitendra, Corroy, & Dupuis, 2013b; Jitendra & Hoff, 1996; van Klinken, 2012).

However, care must be exercised when teaching from a schematic point of view. Elementary students, especially those with learning disabilities, may tend to draw pictorial representations of the word problem, which can pose a variety of
problems: (a) drawing time can be greatly increased if the student is particular about the way their picture looks, (b) students may become so distracted by the drawing process that they lose sight of the problem at hand, (c) hand-drawn visuals, whether schematic or pictorial in nature, may be undecipherable (Foster, 2007; van Garderen, Scheuermann, & Jackson, 2012). It is important to remember, however, that students could increase their chance of success for solving word problems by close to six times using accurately drawn visual schematics. Nevertheless, inaccurate drawings markedly decreased their chances of success (Boonen et al., 2014). These authors pointed out that students used visual representations to help them solve word problems only 35% of time. Although the authors did not determine the reason for this low percentage, they did speculate that inadequate instruction concerning which visual strategy to choose, and how to use it, may be the reason.

Fuchs et al. (2003) produced results from a form of SBI instruction (schema-broadening instruction) that showed promise for general education students. In this study the authors focused on expanding, or broadening, the nature of the defining schema of the problem to facilitate transfer of this skill to other, more novel problems. However, the results of this study for students with disabilities were inconsistent. Interestingly, most studies reviewed on SBI revealed the lack of success to maintain problem-solving skills over time (Jitendra, Dupuis, Rodriguez, Zaslofsky, Slater, & Cozine-Corroy, 2013a).

The Singapore Math approach has shown great success abroad and has refined the visual representations of SBI into easy to understand bar models (Beckmann,
2004; Hoven & Barelick, 2007). This method has been attributed to students in Singapore placing near the top of mathematical problem solvers in the world (Englard, 2010). In her informal study, Englard found that third grade students, after receiving bar model strategy instruction, out-performed other third grade students in control, as well as fourth and fifth grade students outside the study.

As the focus and attention on the importance of teaching real world word problems continues to rise, research has unveiled another challenge in success for students with disabilities, that is, the complexity of the linguistics involved. Reading comprehension in general, and math vocabulary in particular, are significant contributors when addressing word problems.

Teaching the importance and usefulness of math vocabulary must be handled with finesse. As an intervention specialist, it would be easy to fall into the trap of teaching “key words” to drive completion of basic word problems under the guise of success (“if you see the word ‘total,’ you know this is an addition problem”). Caution must be exercised. Relying on this short cut can subvert mathematical understanding of a generalizable process for solving all types of word problems (Beckmann, 2004; Clement & Bernard, 2005; Miller & Mercer, 1993a).

Furthermore, determining exactly which words to use in self-created word problems should be handled with scrutiny. Word problems using context-driven words can enhance the ability of students to draw on previous experiential knowledge (Monroe & Panchyshyn, 2005; Reusser, 2000). This can increase engagement, but as
important, can free up working memory, helping the student to focus on the problem-solving process.

Monroe and Panchyshyn (2005) suggested eight ways teachers can increase word problem success by attending to the impact words and contexts can have on students: (1) teach math vocabulary daily, (2) reduce skill load by providing a few problems, (3) create context with your word problems by using familiar ideas, (4) link word problems to books the class is reading, (5) use enough words to help students create a mental image; (6) link word problems to other content areas (i.e., science), (7) link word problems to students’ experiences, and (8) have students write their own word problems.

Additionally, Miller and Mercer (1993) found that word problem-solving success increased in students with learning disabilities if the complexity of the word problems were taught in a more directly scaffolded fashion. For instance, using word problems consisting of subtraction or multiplication methods, the authors used a graduated word problem sequence (i.e. increasing semantic structure difficulty). This intervention strategy proved successful on elementary students with learning disabilities. In addition, Kaffar (2014b) outlined success in using the graduated lesson sequence approach for elementary students with MD when solving word problems involving addition and subtraction with regrouping.

Attention to the importance of understanding mathematics vocabulary is underscored by Gough (2007) reminded us of the ambiguity of mathematics language—a formal, human constructed language: “mathematics borrows words that
already exist, with everyday meanings, and reshapes or redefines the intended, specialist technical meaning” (p. 12).

A closely related prerequisite to solving word problems is reading ability (Fuchs et al., 2006; Jitendra et al., 2013b; Reusser, 2000). Zheng et al. (2011) showed strong reading skills help overcome deficiencies in WM for students with learning disabilities, thus facilitating successful word problem outcomes. In addition, if technical reading skills were controlled, success on word problems was highly correlated to reading comprehension (Vilenius-Tuohimaa et al., 2008). Comprehension issues related to language use affect not only students with learning disabilities, but also students in low socioeconomic status situations as well as those who were English language learners (Abedi & Lord, 2001). Suggesting the use of reading strategies while addressing mathematics may help the struggling student overcome some of these barriers to comprehension (Foster, 2007).

Specifically, Swanson et al. (2012) showed the effectiveness of a generative learning strategy in terms of working memory use and word problem-solving accuracy. Using paraphrasing techniques (which enhances text comprehension) relevant to proposition use in word problems, these authors showed an increase in problem-solving accuracy, especially among students with MD. They also suggest that these results were mediated by working memory capacity, the demands on which were greater for students with MD.

Paraphrasing interventions were also shown to increase word problem comprehension by restating the question of the problem, as well as the relevant and
irrelevant propositions described in the problem (Moran, Swanson, Gerber, & Fung, 2014). The increased word problem comprehension lead to greater problem-solving accuracy compared to students in the control condition. Moran et al. (2014) speculated the increase in accuracy may be due to the effect paraphrasing has on working memory.

Additionally, a generative strategy instruction approach teaches a more qualitative process, where the students are not overly occupied with immediately generating an answer. Alternatively, the students focus on the quantities involved and how they relate to each other. In this way, they generate a better understanding of the mathematical relationship involved in solving the problem correctly (Clement & Bernard, 2005). Consequently, due to this increase in conceptual knowledge, students are more able to transfer their knowledge of problem-solving to other novel situations.

**Purpose**

The purpose of this study was to explore the effects of using a combination of explicit instruction, cognitive strategies, and graduated lessons on the success of solving word problem situations requiring addition and subtraction with regrouping for elementary students with math difficulties. Literature shows that typical mathematics textbooks do not address how addition and subtraction processes are related (as cited in Jitendra, Haria, Griffin, Leh, Adams, & Kaduvelloor, 2007b). Therefore, four types of word problems are defined and used in this study, which align with the CCSS: Add To, Take From, Put Together, and Compare.
Explicit instruction will employ the main components recognized in the literature as being effective for teaching mathematics: (a) demonstrations and models for solving problems; (b) many examples for solving problems; and (c) multiple practice opportunities. In addition, there will be time for ample student dialogue and teacher feedback.

The cognitive strategies used are mnemonics (acronym) and visual SBI as well as: read; paraphrase; visualize; compute; check; advance organizers; skill modeling; explicit practice; task difficulty control; elaboration; and questioning. For this study, a version of the Singapore bar model for mathematical word problems was used. At the same time, scrutiny was given to the word choice and reading level of the word problems, taking into consideration comorbid reading difficulties in some students. Finally, word problems for students participating in this study were developed using context-driven themes.

By applying these strategies, students who struggle with mathematics may flourish more and feel less “teased and miserable” as Temple Grandin did. This adjustment in competence and success can lead to dramatic changes in a person’s life story.

Research Questions

1. Do students with learning disabilities improve their ability to solve word problems with regrouping after receiving an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence?
2. Do students with learning disabilities improve their computation with regrouping for addition and subtraction after an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence?

3. Do students with learning disabilities report high levels of satisfaction with an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence?

**Definition of Terms**

*Advance Organizer.* This instruction tool prepares the pupil for the day’s lesson: connect to previous lesson; identify what will be taught; provide rationale for lesson (Miller & Mercer, 1993a).

*BBB.* “A mnemonic used for cueing in mathematics strategy instruction Bigger number on Bottom means Break down and trade” (Miller, Kaffar, & Mercer, 2011, p. 117).

*Conceptual Knowledge* is an understanding of an individual of how significant concepts relate to each other. It also involves the ability to apply that knowledge across different systems and novel situations (Robinson & Dube, 2009).

*Cognitive Learning* is the process of adding new knowledge to prior knowledge.

*Cognitive Strategy* is an instructional strategy model that centers on the process involved for solving mathematics problems. Models include: read; paraphrase; visualize; hypothesize; estimate; compute; and check (Montague, 1997). Others are:
advance organizers; skill modeling; explicit practice; task difficulty control; elaboration; task reduction; questioning; and providing strategy cues (Gersten et al., 2009).

**Combined Models** are instructional practices that include specific elements of both explicit and strategic instruction, such as: sequencing; repetition/practice; segmentation; dialogue; difficulty control; modeling; small group; strategy cues (Bryant et al., 2003).

**Describe and Demonstrate (Model).** Here the teacher works through problems with explanations and metacognitive think-alouds (Miller & Mercer, 1993a).

**Explicit Instruction** is direct instruction that provides demonstrations and models for solving problems, many examples for solving problems, multiple practice opportunities and stresses the importance of student dialogue and teacher feedback (National Mathematics Advisory Panel, 2008).

**Extraneous Information** regards extra, irrelevant, or nonessential information added to a word problem (Kaffar, 2014a).

**FAST.** “A mnemonic used for mathematics strategy instruction. Find what you’re solving for. Ask yourself, ‘What are the parts of the problem?’ Set up the numbers. Tie down the sign” (Mercer & Miller, 1992, p. 130).

**GSI** stands for General Strategy Instruction; mathematics instruction typically used in the general education classroom (Jitendra et al., 2007b).

**Generative Strategy Instruction** teaches a qualitative process to solving word problems, where the students are not overly occupied with immediately generating an
answer. Alternatively, the students focus on the quantities involved and how they relate to each other.

*Graduated Word Problem Sequence* involves word problem presentations beginning with one or two words, increasing to phrases, to sentences, and then to paragraphs; initially not containing extraneous information and gradually containing extraneous information. The students end the sequence by developing their own word problems (Miller & Mercer, 1993b).

*Guided Practice.* During this phase of instruction, students work problems with teacher assistance, feedback, and cues, if needed (Miller & Mercer, 1993a).

*Independent Practice.* During this phase of instruction students practice skills independently (Miller & Mercer, 1993a).

*Memory, Long-Term* is where information is stored and retrieved through associations by repetitive exposures (Goodwin, 2014).

*Memory, Short-Term* is where sensory input is first noticed; it remains for about 30 seconds (Goodwin, 2014).

*Memory, Working.* Information stays here about 20 minutes by consciously focusing on stimuli (Goodwin, 2014).

*Mnemonics* are systematic procedures for enhancing learning and memory (Belleza, 1981).

*Mnemonics-Acronym.* A word formed from the initial letters of other words used as a tool to help students solve math word problems (Miller & Mercer, 1993a).
Paraphrasing interventions restate the question of the problem, as well as the relevant and irrelevant propositions described in the problem (Moran et al., 2014).

Procedural Knowledge requires understanding steps to carry out activities or to perform tasks (Miller & Hudson, 2007).

RENAME. “A first letter mnemonic used for mathematics strategy instruction. Read the problem. Examine the ones column. Note ones in the ones column. Address the tens column. Mark tens in the tens column. Examine and note the hundreds; exit with a quick check” (Miller, Kaffar, & Mercer, 2011, p. 117).

SBI stands for schema-based instruction. It focuses on the common underlying structures in word problems that allow them to be solved according to which category, or schema, they belong (Jitendra et al., 2007a).

Think-alouds. Modeling out loud the metacognitive process of solving a problem. This allows the student to know how the instructor thought about the steps required to find a solution.

Word Problems are math problems presented in a linguistic fashion in which arithmetic is employed to solve (Fuchs et al., 2006).
Chapter II

REVIEW OF RELATED LITERATURE

In this chapter the following concepts and strategies pertaining to instruction of addition or subtraction word problems requiring regrouping is discussed: (1) word problem types, (2) explicit instruction, (3) cognitive strategies, (4) graduated lesson sequence, (5) working memory, and (6) linguistics.

Word Problem Types

It is important to note the difference in word problem types used for some studies in the field (Jitendra et al., 2007b) and those used for the study pertaining to this thesis. Both are grounded in the part-part-whole conceptualization for addition- and subtraction-based word problems. Nevertheless, they can be contrasted in the following ways.

To begin, the “change” word problem type employed by Jitendra et al. (2007b) encompasses “add to” and “take from” situations. In the study for this thesis, these two operations are identified as part of their own distinct word problem type. Specifically, “add to” problems consider situations with a beginning value where more of the same are introduced and the participant is asked to find the ending value.
Similarly, “take from” problems are identified as offering a beginning value where some of the same are removed and the participant is asked to find the ending value.

Next, the “combine” (or group) word problem type used by some authors is similar to the more specifically defined “put together/take apart” word problem use for this thesis. Here, two smaller parts are combined to make up a large group. Conversely, the problem may present itself as having a large group value into which two smaller groups can be defined. The value of one of the smaller groups would be given in this type of problem.

Finally, a “compare” word problem type has been defined in which a larger and a smaller value are given and the participant is asked to find the difference. The “compare” word problem type is also regarded in this thesis and is called by the same name.

Importantly, values for any part of these word problems (add to, take from, put together/take apart, and compare) may be given, and any one part may be the unknown. Moreover, the choice in using the four defined word problem types aligns with those four described in detail in the Common Core State Standards for Mathematics, (Common Core State Standards Initiative, 2014a). Since a great majority of states and U.S. territories have already adopted the CCSS, it behooves researchers to henceforth use similar, if not the same, terminology and definitions.
Explicit Instruction

Direct, explicit instruction has become the normal mode of teaching all curricular areas in modern special education research studies. Swanson and Hoskyn (1998) recommended combining explicit instruction with strategy instruction to produce the most beneficial instructional outcomes. The articles summarized and cited herein all include explicit instruction as the base instructional method to which other approaches or strategies have been added. Similarly, the study for this thesis used sound, direct and explicit instruction as a base from which to build the intervention plan. The format for explicit instruction which is used for this thesis is that defined by Miller and Mercer (1993b). Their model includes the following steps: advance organizer, demonstrate and model, guided practice, and independent practice.

Cognitive Strategies

Cognitive strategies are popular and are gaining importance for instruction of students with disabilities. Those relating to the study for this thesis include mnemonics and schema-based instruction (SBI). This section begins with a discussion of the use and benefit of mnemonics, followed by SBI.

Ferreira (2009) investigated the use of explicit instruction embedded with a mnemonics strategy coupled with a concrete-representational-abstract sequence problem-solving approach. In this study, six fifth-grade students with learning disabilities, age 10-12, were formed in two triads. Each group was administered intervention instruction for problems involving subtraction with regrouping and for
word problems following a multiple probe across subjects with one replication design. Results were assessed using pretest and posttest scores.

The lessons included sound, explicit instruction incorporating the following parts: advance organizer; describe and model; guided practice; independent practice; and problem-solving. Furthermore, a CRA sequence was followed throughout the lessons. Beginning with the concrete phase, manipulatives were used to assist the participant in visually interpreting the mathematical operation of subtraction, thereby increasing conceptual reasoning. After a series of lessons at this phase, the lessons progressed to the representational phase as the participants were charged with replacing the use of physical manipulatives with two-dimensional diagrams. This phase helps link procedural knowledge to conceptual knowledge.

Before beginning the final abstract phase, the participants were introduced to the mnemonics FAST, RENAME, and BBB as a learning strategy to aid the transition from representational to abstract computation. The letters of the first mnemonic, FAST, cued the students to: Find what you’re solving for; Ask yourself, “What are the parts of the problem?”; Set up the numbers; and Tie down the sign. Also, the letters of the second mnemonic, RENAME, cued the students to: Read the problem; Examine the ones column: use the BBB sentence for ones; Note the ones in the ones column; Address the tens column: use the BBB sentence for tens; Mark the tens column; and Examine and note hundreds; exit with a quick check. Finally, the letters in BBB cued the students to consider the relative magnitude of the digits being subtracted: Bigger number on Bottom means Break down a ten (hundred) and trade.
The last (abstract) phase of instruction was then introduced with these mnemonic tools in hand to scaffold the participants in successful problem-solving.

Questions Ferriera (2009) asked for this study included:

1. Did the intervention lead to increased performance on subtraction with regrouping lone problems and words problems?
2. Did the intervention lead to increased computational fluency for subtraction problems with regrouping?
3. Did conceptual understanding of this process increase due to the intervention?
4. Did the intervention teach participants to better discriminate between problems with and without regrouping?
5. Did participants maintain subtraction with regrouping skills?
6. Did the participants express satisfaction with this intervention?

Results from pretest to posttest and from survey revealed that, on all accounts, performance on subtraction with regrouping problems increased and satisfaction was high. Likewise, research has shown the use of mnemonics to enhance math performance to be a valid practice (Belleza, 1981; Bryant et al., 2003; Flores et al., 2014; Kaffar, 2014b; Maccini & Ruhl, 2000; Mancl, Miller, & Kennedy, 2012; Mastropieri & Scruggs, 1989; Miller & Kaffar, 2011a; Miller & Kaffar, 2011b).

These results are significant to the study for this thesis because quality explicit instruction will be used along with the mnemonics learning strategy component. Particularly, FAST, RENAME, and BBB will be used throughout the lessons for this
study to assist students in successful word problem computation involving addition and subtraction operations with regrouping.

Ferreira (2009) and others (Flores et al. 2014; Kaffar, 2014b; Mancl et al., 2012; Miller & Hudson, 2007; Miller & Kaffar, 2011b; Miller & Mercer, 1993a; Miller & Mercer, 1993b; Miller, Stringfellow, Kaffar, Ferreira, & Mancl, 2011) also emphasized the development and balance of conceptual (over-all design), procedural (step-by-step process), and declarative (factual spontenaety) knowledge. Research has shown the value of not only the concrete phase of CRA instruction in deepening learner’s conceptual knowledge, but has also shown the validity of the representational phase of mathematics instruction to reinforce conceptual understanding (Gersten et al., 2009; Miller & Kaffar, 2011b).

Explicit instruction was combined with another cognitive strategy, SBI, by Jitendra et al. (2007b). SBI is an approach that teaches how to translate the schema of the word problem in text form to that of a simple diagram, or schematic (Jitendra et al., 2007a). In developing the diagram, considerable attention is given to grasping the conceptual knowledge underlying the problem, that is, how the parts of the problem relate to the whole, for example. Once the proper diagram is created, an accurate mathematical sentence can be developed, which then would rely on proper procedural knowledge.

Jitendra et al. (2007b) tested the efficacy of SBI instruction on 88 third grade students, ages 9 to 11 years, as compared to the traditional general instruction strategy (GSI) method. Six teachers participated in this study.
The design was pretest versus posttest scores and sought to discover the effect of instruction using SBI versus multiple, traditional strategies (generative strategy instruction, GSI) on the success of solving word problems. In addition, the study also looked at maintenance of problem solving skills over time, transfer effects of the strategy, and the influence of word problem-solving instruction on computational skills.

Addition and subtraction word problem types were defined as change, combine (or group), and compare types. SBI related to these word problem types considered the part-part-whole approach to solve the problem. For example, a change problem is defined as having a beginning value, a number to change that beginning value (either increase or decrease), and an ending value. While a combine (or group) type problem involves two or more smaller groups, or parts, that when combined (or added) yields a large group, or whole (sum). Finally, compare type word problems were characterized as discriminating between a larger and smaller component, and finding the difference between the two.

Results from the study showed students receiving SBI instruction outperformed those receiving GSI in all areas. These results are significant to this thesis in a number of ways. First, an abbreviated form of SBI will be used to help determine which solving strategy to use. The schematic design used is modeled after the Singapore Math approach and will be discussed in more detail further in this chapter. In the Jitendra et al. (2007b) study, the schemata were faded out of instruction during intervention as will the drawings used in the study for this thesis.
Second, improved computation skills were realized after the word problem intervention in Jitendra et al. (2007b). In the study for this thesis, addition and subtraction with regrouping computational skills will also be monitored via pretest and posttest to determine if skills improve in this area.

Jitendra et al. (2007b) noted limitations to their study of small sample size and distracting learning environment (building construction). Although the sample size is limited in the study for this thesis, the learning environment is improved from the Jitendra et al. study in that all groups are taught in a separate resource room, away from major distractions.

Griffin and Jitendra (2008) followed and extended the work of Jitendra et al. (2007b) by examining SBI instruction versus GSI (multiple strategy) instruction taught by general educators. These authors worked with mixed-ability groups comprised of a total of 60 third grade students, age range 8-10. By ordering performance on a nationally-normed mathematics assessment, the authors divided the learning groups into reasonably equitable mixed-ability groups—two groups receiving SBI instruction and two receiving GSI instruction.

Griffin and Jitendra (2008) used a between-subjects, pretest-to-posttest-to-delayed-posttest group design to investigate the following questions:

1. How do SBI and GSI instructional methods for problem solving compare when administered to mixed-ability participants?

2. Would the effects of SBI instruction hold up over time?
3. Would change in performance over time differ for the two groups of participants?

4. Would there be an effect on computational skill ability due to word problem-solving instruction?

Word problems were defined as those categorized in the Jitendra et al. (2007b) study: change, group, and compare. However, two-step problems were also considered. In addition, paired partner work was initiated as part of the guided practice portion of instruction. Results revealed that instruction with both SBI and GSI produced improvement in accuracy for word problem-solving and computation skills. The authors identified the length of the learning sessions (100 minutes each) as having a mediating effect on instruction. In addition, the authors did not consider linguistics in terms of reading comprehension in this study and postulate varying levels within the participant population may also have influenced their results.

Griffin and Jitendra (2008) discussed implications for future research which would apply strategy instruction, coupled with quality explicit instruction for word problems with distinct, like-ability participants. For example, they questioned how such instruction would affect students with specific math difficulties. The study for this thesis will administer its intervention instruction on elementary students identified as having math difficulties and receiving small-group instruction for mathematics. This may offer insight to how well the population receives explicit and strategy instruction for word problems.
In their study, Griffin and Jitendra (2008) faded the use of schematic diagrams over time. Nonetheless, students could rely on short-handed versions of the diagrams to aid in successful problem-solving. An abbreviated version of bar diagrams is used for this study to assist participants in visualizing the numerical relationships presented in a textual format. Focus is emphasized more with the numerical relationships within the word problem which lead to the proper mathematical sentence and solution, rather than on the bar diagram itself.

SBI has shown promise in other studies such as addition word problems for third-grade students with math difficulties (Jitendra et al., 2013a), adding SBI to response cards for students with learning disabilities to solve word problems (Schwab, Tucci, & Jolivette, 2013), and providing SBI instruction to a student with autism (Rockwell, Griffin, & Jones, 2011). Additionally, the Institute of Educational Services (IES) Practice Guide, “Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools” as cited on the What Works Clearinghouse website states in Recommendation 5: “Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas” (Gersten et al., 2009, p. 30). The panel went on to express how “critical” this aspect of word problem instruction is (Gersten et al., 2009, p. 30). However, it is important to keep the drawings simplistic (Foster, 2007; van Garderen et al., 2012).
One method for representing word problems using simple sketches is the Singapore bar model. The Singapore approach has been used for years overseas with great success and has gained some popularity in the U.S. (Beckmann, 2004). Since the country of Singapore continues to have a stronghold in the number one position for highest average math score among 54 nations participating in the 2011 Trends in International Mathematics and Science Study (TIMSS), National Center for Education Statistics [NCES], 2011), researchers feel their bar model approach to solving math problems is worth taking a look at (Beckmann, 2004; Englard, 2010; Hoven & Barelick, 2007).

For one-step addition or subtraction word problems, a total of three bars can be sketched which align with the compare word problem model (Beckmann, 2004). One bar (the total amount for the word problem situation) should be as big as the two parts which comprise it. By successfully attributing the values given in the word problem, the learner can develop the corresponding math expression using the correct operation. In this way, the learner focuses on the relationship between the values given in the problem, thus enhancing their conceptual knowledge. From there, procedural knowledge is strengthened while the answer is derived.

Similar approaches with slight alterations can easily be used for part-whole problems as well (Englard, 2010). A simplified version of the Singapore bar model is used in the study for this thesis. The three bars (one large; two small) are used for all four word problem types. In this way, the students only have to remember one version
of the bar diagram so the transition from text to abstract numeracy is made more easily. As proficiency in this transition grew, the use of the drawings was faded out.

**Graduated Lesson Sequence**

The study by Miller and Mercer (1993) introduced a graduated lesson sequence to promote success of solving mathematical word problems for elementary students with learning disabilities. This approach was coupled with a concrete, semi-concrete, abstract learning strategy. Using a pretest-intervention-posttest design, Miller and Mercer field-tested their lesson design for multiplication and subtraction word problems on two different occasions.

At first, graduated word problem lessons were presented by using simple words, then phrases, then sentences, and finally paragraphs to format word problems for a specific mathematical operation.

The word problems presented were very easy at first, and increased in difficulty as the lessons progressed. After mastery of the paragraph format, extraneous information was added to the word problem. Finally, students were required to develop their own word problems. The authors applied this process for multiplication and subtraction word problems. The results for both operations on posttest showed improved success in word problem computation. Teacher satisfaction was high for the lesson sets of both mathematical operations as well.

The first study included eight female teachers and 54 (38 male and 16 female) students in ages ranging from 9 to 12. These students were also identified with LD in
mathematics. The students were not able to multiply basic facts. A mean posttest score of 94% on word problems was obtained from this group of students. They also achieved a mean posttest score of 92% on multiplication computation problems.

The second study involved four female teachers and 13 students, ages from 7 to 9 years old with LD in mathematics. This study investigated subtraction problems. These students could add facts up to 9, but were not successful with subtraction. A mean posttest score of 92% was achieved for word problems. For simple subtraction computation, the mean posttest score was 95%. These results show dramatic improvement in participants for both multiplication and subtraction operations. Results are similar for research in multiplication and subtraction (Mercer & Miller, 1992), addition with regrouping (Carmack, 2011; Miller & Kaffar, 2011b), subtraction with regrouping (Flores et al., 2014), subtraction (Ferreira, 2009), addition and subtraction with regrouping (Kaffar, 2014b), and subtraction with regrouping (Mancl et al. 2012; Miller, Kaffar, & Mercer, 2011).

The work of Miller and Mercer (1993) is important to the study for this thesis because it shows success of using a graduated word problem sequence. This approach will be woven throughout explicit instruction lessons presented to students as they learn how to compute four types of word problem situations using addition and subtraction with regrouping. However, the word problems for this study begin as simple sentences for each word problem type and are presented separately. After mastery, the word problem for the same problem type increases in difficulty with the addition of extraneous material and multiple regrouping opportunities. As one
problem type is mastered on both these levels, the next problem type is introduced, again with a simple sentence format, progressing to the more complicated format. The design continues until all four-word problem types have been mastered. Finally, all problem types are presented randomly at varying levels of difficulty to test identification and generalization of the solving method.

A graduated word problem sequence naturally fits the scaffolded teaching design typically used throughout the field of special education. Since word problems have traditionally been seen as a troublesome area to learn for students with learning disabilities, and since there has been a renewed focus on the mastery of word problems throughout the CCSS, it would benefit students and schools alike to increase achievement in this area.

Another area of difference between the aforementioned studies and that used for this research is the previous studies focused instruction on the mathematical operation alone. They addressed related word problems in a mini-lesson at the end of each teaching sequence. The study for this thesis is dedicated to teach word problems as the main focus of instruction. However, computational fluency problems are provided at the end of each of these lessons. Research is lacking in this area of instruction; therefore, results will add meaningful data to the growing pool of mathematics instruction data for students with learning difficulties.

It is also important to note that the study for this thesis and the work of Mercer and Miller and the others differ in that they used the CRA approach to learning word problems while this study will not. The rationale revolves around the perceived math
experience of the learners. By the time the participants are ready for word problem study, it is presumed they have already been exposed to instruction for addition and subtraction with regrouping, whether CRA for those lessons was used or not. This is not to state, however, that the participants necessarily have mastered the skills necessary for successful addition or subtraction with regrouping.

Additionally, declarative knowledge, that of supplying mathematical answers with efficiency and fluidity, may take some time to develop and can require a substantial amount of practice. To this end, Miller and Kaffar (2011b) purported moving on to the next math concept while the learner continues to practice and master declarative knowledge of previously-learned skills. Built into the study for this thesis are opportunities for continuing practice on the addition and subtraction with regrouping procedures whilst applying those skills to more advanced concepts such as word problems.

**Working Memory**

Recent research regarding the moderating effects of WMC on strategy instruction which taps into the use of the visual-spatial sketchpad portion of WM, such as a visual bar model, has been conducted by Swanson (2014). In his study, 147 third grade public school children (74 female, 73 male) from the southwestern U.S. were randomly placed into one of four word problem instructional groups: (a) verbal strategy intervention, (b) visual strategy intervention, (c) verbal plus visual strategy intervention, or (d) control (no intervention). Of these 147 participants, 59 were
identified as at-risk for math difficulties (MD) (based on the 25th percentile cutoff score on standardized achievement measures). There were 88 students without MD.

The design was pretest-posttest in nature and the interventions consisted of 20 scripted lessons given in 8 weeks. These lessons were broken into four phases: (1) warm-up phase (calculations and puzzles), (2) instruction phase (teach particular strategy), (3) guided practice (students working problems with feedback, and (4) independent practice (students working problems without feedback.)

Classification measures considered within this research included: fluid intelligence; word problem solving ability; reading skills measures consisting of word recognition, reading comprehension, and arithmetic calculation; and working memory capacity measures of conceptual span task, sentence/digit span, and updating. A mixed analysis of covariance (ANCOVA) statistical model was used to analyze the effect of the treatment interventions. Swanson (2014) found that students with higher WMC benefited substantially more by using cognitive strategies such as the bar model, than students with relatively lower WMC.

Therefore, in the study for this thesis, the choice to use the bar model tool to solve word problems was left up to each individual participant. Consequently, if the student felt using the bar model increased positive outcomes on the word problems, then they could use it. However, if the student felt frustrated in using the bar model to determine the correct mathematical operation to use, they could choose not to draw the diagrams.
Linguistics

Another factor in determining how well a student can solve word problems was investigated in a study by Vilenius-Tuohimaa et al. (2008). They found an important part of linguistics, reading comprehension, plays a role in the success of such computations. In this study, the authors used 225 fourth-grade students (107 girls, 118 boys) from a demographically universal Finnish school district. Ability levels for these students were considered mixed. 24.4% of the students in this study received special education services (although not all were on an IEP).

The students were screened according to reading ability: poor readers (PR) group and good readers (GR) group. Additionally, they were assessed according to word problem-solving ability in a probe of 20 problems. An analysis of variance (ANOVA) was conducted on the scores and showed statistically significant differences ($p < .001$) between the groups. The GR group performed better than the PR group on solving math word problems. This result remained true after controlling for technical reading ability. In fact, further statistical analysis indicated a strong relationship between reading comprehension and word problem-solving; however, overall reading comprehension ability and technical reading skills both were shown to play a role in the level of success of solving mathematical word problems.

Furthermore, in the 2012 meta-analysis conducted by Zheng, Flynn and Swanson, reading ability was found to impact the success level of word problem-solving for students identified as having math disabilities. This result held true for students with math disabilities both in treatment and in control as outperforming those
students with math disabilities and reading difficulties in treatment and control. These results were also shown to be true in earlier meta-analyses (Baker, Gersten, & Dae-Sik, 2002; Swanson & Hoskyn, 1998).

Along with reading comprehension, other linguistic factors have been shown to affect word problem outcomes in the form of sentence complexity and semantics, especially for English language learners (ELL), students from low socio-economic status (SES), and students with learning difficulties (Abedi & Lord, 2001). These results were obtained from a study comprising 1,174 eighth-grade students (54% boys, 46% girls) from 11 schools in the Los Angeles area. The math test components were derived from 20 carefully selected items from the National Assessment of Educational Progress (NAEP) mathematics assessment. One-half was linguistically modified to easier formats and was paired with the other ten unaltered problems for test booklet A. Test booklet B was created by modifying the alternate 10 problems (those in original format in booklet A) and adding the previously altered problems from booklet A in their original, unaltered format.

Statistical analysis of results showed, in part, that lower achieving students in math performed better on the linguistically modified math tests, yielding the highest percentage improvement based on gain score of 6.7% than control, ELL, or low SES categories. These results are substantiated with those pertaining to the study by Zheng et al. (2011) where reading ability was found to mediate WM with respect to mathematical solution accuracy. Specifically, the authors found effects on the central
executive and phonological loop portions of WM with respect to problem-solving outcomes.

It is important to contrast these types of linguistic considerations for word problems with that of the traditional “key word” approach to solving word problems. Solving word problems based on identifying “key words” can sometimes yield a correct answer. However, not only are these results inconsistent, this approach can undermine the mathematical meaning of the problem, thereby interfering with contextual and procedural understanding (Clement & Bernard, 2005). Using key words can also result in impeding success at attempts to generalize word problems. For example, if students identify the word “altogether” as a signal for addition, they would find frustration in obtaining the correct result for the following word problem: “Tim had 10 baseballs altogether. He got some for his birthday and three from his father. How many baseballs did Tim get for his birthday?”

The key-word approach will be avoided in the study for this thesis to minimize confusion and increase accuracy. It may be necessary to “unteach” this method as the use of key words relative to word problems is suspected as being widespread.

Linguistics is an important consideration for this study because the research questions for it relate to mathematical solving ability alone and do not include reading ability considerations. Therefore, the Flesch-Kincaid (Kincaid, 1975) reading level of the word problems used throughout this study were kept at a third grade reading level for the participants who were in grades four and five. In addition, if a student was experiencing difficulty reading the problems, the problems would be read aloud. In
this way, the effect of reading ability on the outcome of the study would be minimized.
Chapter III

METHODOLOGY

Introduction

The purpose of this study was to investigate the effects of combined-strategy instruction on performance of word problem computation success involving addition and subtraction with regrouping for students with learning disabilities. This chapter addresses methodology and related questions in the following framework: (a) research questions, (b) participants, (c) setting, (d) instrumentation, (e) materials and equipment, (f) design, (g) treatment, (h) interscorer reliability, (i) fidelity of treatment, and (j) treatment of data.

Research Questions

The following research questions have been answered in this study:

1. Do students with learning disabilities improve their ability to solve word problems with regrouping after receiving an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence?

2. Do students with learning disabilities improve their computation with regrouping for addition and subtraction after an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence?
3. Do students with learning disabilities report high levels of satisfaction with an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence?

Participants

A total of seven students with learning disabilities in grades 4 and 5 participated in this study. The age range for these participants was from 9 years, 5 months to 11 years, 4 months. One female and six males comprised the group, and of these, six were White, Non-Hispanic and one was Black.

Participation pool. A convenience sample was used to select the participants. These students were enrolled at one publically funded elementary school located in a midwestern small town. The pool of participants consisted of students who qualify for direct special education services in the area of mathematics instruction. Three different licensed teachers in special education managed the caseloads of these students. See Table 1 for a participant demographic summary.
Table 1

Participant Demographic Data

<table>
<thead>
<tr>
<th>Number</th>
<th>Group</th>
<th>Gender</th>
<th>Ethnicity</th>
<th>Age</th>
<th>Grade</th>
<th>Disability</th>
<th>Intelligence SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A-1 Yellow</td>
<td>M</td>
<td>WNH</td>
<td>10.7</td>
<td>5</td>
<td>OHI</td>
<td>82</td>
</tr>
<tr>
<td>2</td>
<td>A-2 Yellow</td>
<td>M</td>
<td>BLA</td>
<td>11.1</td>
<td>5</td>
<td>OHI/SLD</td>
<td>82</td>
</tr>
<tr>
<td>3</td>
<td>A-3 Yellow</td>
<td>F</td>
<td>WNH</td>
<td>11.4</td>
<td>5</td>
<td>SLD/SLI</td>
<td>84</td>
</tr>
<tr>
<td>4</td>
<td>B-2 Green</td>
<td>M</td>
<td>WNH</td>
<td>10.1</td>
<td>4</td>
<td>OHI/SLD</td>
<td>88</td>
</tr>
<tr>
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<td>WNH</td>
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<td>4</td>
<td>OHI/SLD</td>
<td>75</td>
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<td>5</td>
<td>ASD/SLI</td>
<td>74</td>
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<td>M</td>
<td>WNH</td>
<td>10.7</td>
<td>4</td>
<td>SLD</td>
<td>82</td>
</tr>
</tbody>
</table>

F: Female; M: Male
WNH: White, Non-Hispanic; B: Black/African American, Non-Hispanic;
SLD: Specific Learning Disability; OHI: Other Health Disability; SLI: Speech/Language Impaired;
ASD: Autism Spectrum Disorder
Intelligence Assessment: Wechsler Intelligence Scale for Children-Fourth Edition (Standard Score)

Participation selection. Each participant was required to meet specific criteria to be eligible for this study. The participants must have: (a) met the state of Minnesota eligibility requirements to receive special education services; and (b) been enrolled in grade 4 or 5. Also, the parent or guardian of each participant must have submitted a signed informed consent (Appendix A). The student participants must also have submitted a signed informed assent (Appendix B).

Instructional group formations. Dyads and a triad were formed with consideration of grade level schedules and appointed times for typical mathematics instruction. This resulted in groups of students listed in Table 2.
Table 2
Dyads and Triad Grouping of Participants

<table>
<thead>
<tr>
<th>PARTICIPANT</th>
<th>GROUP</th>
<th>DYAD OR TRIAD</th>
<th>CODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>1</td>
<td>A-1</td>
</tr>
<tr>
<td>2</td>
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<td>B-3</td>
</tr>
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<td>1</td>
<td>C-1</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
<td>2</td>
<td>C-2</td>
</tr>
</tbody>
</table>

Setting

This study was conducted within a small-town, neighborhood, K-5 public elementary school in a midwestern town with a population of 12,898. The school has a student population of 935 students and is serviced by 38 grade level teachers, five teacher specialists (e.g., music, media, art, physical education), and 14 teachers who service unique populations of students (e.g., special education, English language learners). Further demographic information about this school includes the following: (a) 3% of the student population is Asian/Pacific Islander, (b) 4.55% of the student population is Black/African American, (c) 1.38% of the student population is Hispanic, (d) 1.38% of the student population is Native American, (e) 89.94% of the student population is White, and (f) 7.93% of the student population lives below the poverty level. This school is part of a district, which is considered small-town rural. The district services approximately 3,976 students, 33% of which qualify for free or
reduced lunch. Of the almost 4,000 students, about 11% are identified as students with disabilities.

Instrumentation

Two curriculum-based assessments (CBA), curriculum-based measurement, eight baseline probes, and a participation satisfaction survey were used for this study. A description of each is detailed in this section.

Word problem pretest and posttest. The first CBA, the *Addition and Subtraction with Regrouping Word Problems Pretest* (Appendix C) included 10 word problems in all: two Add-To problems, two Take From problems, two Put Together problems, and four Compare problems. Also, four of the word problems required addition with regrouping, and six required subtraction with regrouping. Specifically, the problems varied in one addition and subtraction regrouping, two addition and subtraction regroupings, and addition and subtraction with regrouping from zeros involved in one regrouping situation. The numbers embedded within the word problems were comprised of three digits.

Furthermore, all four types of word problems used for the study were randomly represented throughout the pretest: Add To, Take From, Put Together, and Compare. Random problems included extraneous material, as well. This pretest was untimed.

Each problem was scored out of a possible five points. Each point corresponded to using the mnemonic **FAST RENAME** to solve the problem: (1) **F** Correct label for answer; (2) **A** Correct number identification; (3) **S** Correct equation
set-up; (4) T Correct operation identification; and (5) RENAME Correct computation. See Appendix D for a word problem scoring rubric. Scores were calculated as a percentage correct out of 50 points.

The intent of the pretest was to determine the level of word problem-solving proficiency of each participant prior to intervention. Students receiving less than 80% on this assessment showed lack of proficiency in this skill. The same format was used for the posttest assessment. Students scoring 80% or above on posttest were considered to have mastered the skill of solving addition with regrouping and subtraction with regrouping word problems.

Computation pretest and posttest. The second CBA used for this study was the Addition and Subtraction with Regrouping Computation Pretest (Appendix E). This pretest was comprised of 20 three-digit problems, half of which involved addition and half of which were subtraction. Thirteen problems required single regrouping, while seven were double regrouping situations. Each problem was scored as either correct or incorrect and the results were reported as percentage correct out of 20. The posttest was designed and implemented in the same fashion; however, it was comprised of 14 single and six double regrouping situations.

The purpose of this pretest was to reveal the participants’ skill level for solving problems involving computation only. The Addition and Subtraction with Regrouping Computation Pretest was untimed. Similar to the word problem pretest and posttest, mastery of this skill is achieved when the student scores 80% or better.
Computation fluency pretest and posttest. Curriculum-based measurement (CBM) was used to track the change in computation fluency before and after intervention. Here, the *Addition and Subtraction with Regrouping Computation Fluency Pretest* (Appendix F) was administered to all students prior to intervention. Sixteen, three-digit problems were presented. Following standard practice for curriculum-based measurement as outlined in Hosp and Hosp (2003), scores were represented as number of correct digits per minute (cdpm). For example, if the answer to one problem was 421 (correct digits for the problem is three), but the student wrote 420, two correct digits would be awarded for this problem and added to the total correct digits the student earned on all problems answered for the test. To determine the cdpm, the total correct digits earned in two minutes would be divided by two. Half were addition situations, and half involved subtraction. Of the 20 problems, 12 problems included single regrouping and four problems required two regrouping scenarios. This test was timed for two minutes and scores were reported as correct digits per minute (cdpm).

Likewise, the *Addition and Subtraction with Regrouping Computation Fluency Posttest* was designed in a similar fashion to the pretest, except the problems were presented in a randomized order from that which was depicted on the pretest to minimize testing effect (Roediger & Karpicke, 2006).

Word problems with regrouping progress chart. After being presented with scores from the pretests, each student was given a *Word Problems with Regrouping*
Progress Chart (Appendix G) displaying two charting areas: (a) Problem-Solving Graph, and (b) Computation Graph. Both graphs were labeled for receiving results during each step of instruction from pretest through every lesson, including posttest results. In this way, students would receive immediate feedback on the level of success for each phase of instruction they had just completed, following best practices for explicit instruction. This progress chart was designed after that used by Miller et al. (2011b).

**Learning contract.** After recording pretest scores, participants were given a Learning Contract with a description of the commitment they would make to the intervention and a place for them to sign. Likewise, it included a description of the commitment the instructor would make to the lessons and also a place to sign. (See Appendix H for an example of the contract.) Similar Learning Contracts were used by Miller et al. (2011b).

**Word problem baseline probes.** Baselines for word problem-solving proficiency were established by administering four-problem probes at 1-week intervals prior to intervention (Appendix I). The numbers embedded within the word problems were three digits. All four types of word problems used for this study were randomly represented throughout the probes: Add To, Take From, Put Together, and Compare. Additionally, the complexity of the number portion of the problems varied with: addition and subtraction with regrouping from ones to tens or tens to hundreds, zeros
involved in one regrouping situation, and zeros involved in both regrouping situations. Random problems included extraneous material, as well.

The same scoring rubric was used for these probes (Appendix D). The maximum possible score for each probe was 20 points and scores were reported as percentage correct. After three or more probes were administered, intervention could begin.

**Intervention probes.** Each of the fourteen lessons included a learning sheet the students were required to complete with a level of 80% proficiency or better before moving on to the next lesson. For Lessons 1A and 1B, methodology for successfully solving addition and subtraction with regrouping problems involving two, three-digit addends or subtrahends was presented.

Each Learning Sheet (1A and 1B) was comprised of nine total problems: three for the describe and model phase of instruction, three for guided practice, and three for independent practice. As the lesson was scaffolded from teacher-led model and think-alouds, to student-led completion of the problems, the participants were computing their own answers by problem number 5. Therefore, problems 5 through 9 were used to score for mastery. Lesson 1A problems involved addition with regrouping only, while Lesson 1B problems involved subtraction with regrouping only.

Lesson 2 involved learning the procedure for transferring mathematical information from a text format to a numerical equation representation. Therefore, Learning Sheet 2 was comprised of seven word problems: two for the describe and model phase of instruction, two for guided practice, and three for the independent
practice phase of instruction. The numbers embedded within the word problem consisted of 3-digits. Instruction was again, scaffolded to lead participants in computing their own answers by question 4. Consequently, questions 4 through 7 were scored to determine mastery of this skill.

Each problem was worth 5 points, one for each of the following attributes: identification of the correct label, identification of the correct number parts, set-up of the numbers, determining the correct mathematical operation to use, and computing the correct answer.

The remaining lessons used Learning Sheets that followed a combined format of Learning Sheets 1A, 1B and 2. Each included seven word problems as described for Learning Sheet 2, and five computation problems as described for Lessons 1A and 1B. Addition and Subtraction operations for the computation problems were randomly presented throughout the Learning Sheets. The mathematical operation used for the word problems on each Learning Sheet was a function of the type of word problem that was the focus of the lesson (Appendix J for an example of this type of Learning Sheet).

Two scores were obtained from Learning Sheets 3-12: one for solving word problems (word problems 4-7) and one for computation proficiency alone (problems 8-12). The word problems were scored following the same 5-point rubric described for Lesson 2. The computation problems were scored as either correct or incorrect. Both scores were reported as percentage correct.
Throughout all lessons, careful consideration was given to creating context participants could relate to. For example, in discussing Add-To word problems, the PI first began by taking a stack of paper on the table and adding more to the pile. The concept was depicted on the board using a sketch of a bar model and the same language as that used while manipulating the piles of paper.

The language of the word problems on the learning sheets also was chosen not only with readability in mind, but also with subjects students would be more likely to relate to. Such examples of word problem context include: beans, running laps, baseball cards, and cups.

Another way context was used to help the students relate to the word problems and, intervention experience in general, was by referring to their individual alphanumeric assignments (created to enhance anonymity) as “secret codes.” The students liked labeling each sheet with their “secret code” every day.

Creating context was also accomplished by applying a catch-phrase to one of the steps defined by the mnemonic RENAME. One of the parts to the last E in RENAME is to “exit with a quick check,” meaning to solve a complement equation using the opposite sign to check their work. This became a fun expectation as the PI would ask students if they were a “happy camper?” meaning, “Did the quick check work agree with the first calculation?” It was evident students enjoyed this reference as many would draw smiles or stars on their papers every time a problem checked out. Others would look up after completing problems, smiling, and say, “I’m a happy camper!”
Finally, context was enhanced by relating what we did in class to a popular event. For example, while introducing the idea of extraneous material in Lesson 6, the PI told them this was like a big distraction included to try and get them away from what was important in the problem. The popular movie “UP” was referenced using the scene when the dog was trying to tell his master how much he meant to him, and he was distracted in the middle of his heartfelt discussion by seeing a squirrel. From then on, when extraneous material was discovered within a word problem, it would not be unusual for one or more participants to shout out, “Squirrel!” much like the dog did in the movie “UP.” Squirrel became of form of a mascot for the learning groups for the remainder of the lessons.

Satisfaction questionnaire. A satisfaction questionnaire was given to the participants and the end of the study to determine how they felt about the various parts of the intervention lessons and learning to solve word problems involving regrouping (Appendix K). Ten questions comprised the questionnaire and included two answer options: a smile for “yes,” and a frown for “no.” The participants were instructed to select the face that most closely represented their feelings about the question. They were asked to either circle the entire face, or fill in the little dot under the face they chose. All questions were read aloud to the participants.

Materials and Equipment

The Lessons given throughout this intervention were crafted after those developed by Miller et al. (2011b) and represented sound, pedagogical instruction
including: advance organizer, describe and model, guided practice, and independent practice. Descriptions of materials used to enhance instruction follows in this section.

RENAMED reference cards. To aid the students in learning the mnemonic RENAME and the corresponding reminder sentences for subtraction and addition with regrouping, students were given colored cardstock upon which each step of RENAME and its description was written. For Subtraction with Regrouping, the BBB sentences were also included for breaking down the tens and hundreds (Appendix L). For Addition with Regrouping, the “10 or More” sentences were included for regrouping the carryover digit to the next column for the tens and hundreds (Appendix M). These cards were handed out and were the focus for Lessons 1A and 1B.

FAST RENAME reference card. Students were presented with another colored cardstock sheet with the mnemonic FAST written out, as well as RENAME for subtraction and addition with regrouping (Appendix N). The card was introduced in Lesson 2 to assist participants in transferring mathematical information in text form to numerical equations. The students kept the card for the duration of the intervention lessons for easy reference.

Extraneous material reference card. Lesson 6 introduced extraneous material for the first time in the word problem. To help students understand and remember the word and its meaning, a colored cardstock sheet was given to them explaining the word. It also included common synonym. (Appendix O).
Design

A multiple probe design across subjects with two replications was used in this study (Ferreira, 2009; Horner & Baer, 1978). Replicating the conditions by extending baselines to different degrees across groups strengthens the internal validity of the design. It also diminishes what would otherwise be considered threats to the internal validity such as ambiguous temporal precedence or history considerations (Kratochwill et al., 2010). In addition, the baseline condition acts as the individual’s control condition which is an important consideration because there was no control group defined for this study. Replicating both baseline and intervention situations can enhance evidence credibility (Kratochwill et al., 2010). Moreover, conducting three repetitions is considered a requirement to meet single-case design standards set forth by Horner, Carr, Halle, McGee, Odom, & Wolery (2005).

Design conditions (phases) included establishing baseline and administering intervention lessons. This study used three groups, each comprised of two to three students from either the fourth or fifth grade. Each subject and their parent granted permission by signing the Parent Consent and Student Assent forms.

Baseline phase. The study began with establishing baseline conditions from scores received on the Word Problem Baseline Probes. These data were indicative of the level of pre-instructional skills each participant had related to solving word problems with addition and subtraction with regrouping. The probes were given to participants until a stable baseline behavior was indicated or after administering three
probes. Once baseline was established, intervention lessons began for Group A. Typical of multiple baseline studies, Groups B and C continued receiving baseline probes once a week until it was time for that group to begin intervention.

**Intervention phase.** After three baseline probes were administered, Group A began receiving intervention lessons.

Upon completion of Lessons 1A, 1B, 2, and 3, the next group of students (Group B) began their intervention instruction with Lesson 1A, while Group A continued on with the series of lessons. Similarly, when all participants in Group B had successfully completed Lesson 3, the final group of students (Group C) began their intervention instruction with Lesson 1A.

**Satisfaction survey.** After the students completed the posttests of the study, each participant was asked to complete a Satisfaction Survey. The information from the survey indicates how comfortable the students felt with the instruction during intervention, and whether they felt it helped them understand and solve word problems with addition and subtraction. Results also gave an indication of how the participants felt future students would benefit from receiving this instruction.

**Treatment**

The lessons used in this study reflect sound explicit instruction involving the following components: (a) advance organizer, (b) describe and model, (c) guided practice, and (d) independent practice. Furthermore, into all lessons were woven opportunities for teacher think alouds, especially during the describe and model phase
of the lesson; positive, yet constructive and immediate feedback, and rich dialogue which helped to relate the context of the word problem to the students’ prior knowledge.

Additionally, approaches to solving the following four different types of word problems were discussed: (1) Add To, (2) Take From, (3) Put Together, and (4) Compare. Using a graduated lesson sequence, students were required to master individual word problem types while increased difficulty was scaffolded in through varying regrouping situations, extraneous material, and generalization opportunities. The graduated lesson sequence can be found in Appendix P. Studies have shown this graduated approach to be successful in teaching computation and problem-solving skills (Carmack, 2011; Ferreira, 2009; Flores et al. 2014; Mancl et al., 2012; Miller et al., 2011a; Miller & Kaffar, 2011a; Miller & Kaffar, 2011b; Montague, 1997).

Another embedded strategy was the cognitive strategy of mnemonics. FAST, RENAME and BBB were used to support the students in properly working their way through the word problem in an organized, step-wise fashion using easy-to-remember acronyms. A complementary schema-based instruction was also employed to assist the students in identifying the unique qualities of each word problem–its schema--using a bar model graphic similar to those used in the Singapore Bar Model technique. This learning tool was faded out as the lessons progressed.

Each lesson included on-going monitoring in the form of Learning Sheet Probes. Students were required to obtain at least 80% mastery on the word problems portion of these probes before subsequent lessons would begin. If a student did not
achieve 80% mastery, the lesson was retaught and the student completed another Learning Sheet Probe until 80% mastery or better was attained. At this point, the next sequential lesson was given. Students recorded their own Learning Sheet scores on their chart before progressing to the next lesson. This type of self-monitoring helped to create intrinsic motivation to either maintain their high level of achievement, or to work harder to improve their achievement, lesson by lesson.

Mindful of the impact linguistic complexity can have on students who may struggle with reading and comprehension; particular consideration was given to this variable. Each word problem used for the intervention was carefully crafted by paying express attention to reading level and context. To promote valid assessment on the students’ ability to solve word problems, and not have interfering effects of reading challenges, word difficulty was kept to a mid-third grade level, about 3.5 on the Flesch-Kincaid readability scale (Readability-Score.com, 2015)

After students completed the last lesson, three posttests were administered to determine the level of improvement resulting from intervention in the following areas: word problem computation, subtraction and addition with regrouping skills, and subtraction and addition with regrouping fluency. Scores from these posttests were shared with the participants and they graphed them on their progress chart.

Following this, the students were given a satisfaction survey with 10 questions on it relating to their overall opinion of the intervention. The questions were read aloud to the participants, and they chose between “yes” and “no” answers depicted by smiles and frowns, respectively.
Interscorer Reliability

An outside consultant to the study determined interscorer reliability. Twenty percent of randomly selected probes and tests were independently scored per guidelines stipulated in Kratochwill et al. (2010). Interscorer reliability was then determined by comparing scoring results with those of the primary researcher for the study. A percentage of agreements in scoring outcomes between the two were then computed following the formula: agreements (agreements + disagreements) X 100 (Horner et al., 2005).

Fidelity of Treatment

The supervising professor for this study observed 31% of the intervention sessions to ensure the groups received content and instructional procedures with fidelity (Horner et al., 2005). A Fidelity of Treatment Checklist was developed for this purpose (Appendix Q). The particular steps noted on the checklist include: advance organizer; describe and model; guided practice; independent practice; computation practice; and score and provide feedback. Percentage of complete explicit instruction steps was reported.

Treatment of Data

Visual analysis. Visual analysis of results from Baseline Probes, Intervention Probes, Pretests, Posttests, and Satisfaction Questionnaires were used to assess the effects of solving word problems involving addition and subtraction with regrouping.
The results were graphed following multiple probe design guidelines, which strengthen internal validity (Barlow & Hersen, 1984). Visual inspection of level, trend, and variability for each participant’s results were then assessed (Horner et al., 2005).

Level is the first criterion of visual analysis of data and is defined as using the mean score within a particular phase (Kratochwill et al., 2010) to compare the level between baseline and intervention. The second criterion of visual inspection is trend. This is defined as the slope of the best-fit line within a phase (Kratochwill et al., 2010). The stability of the trend line was also inspected visually. The more stable the slope, the more reliable the trend lines were considered. It follows that the intervention was considered more successful when less variability was shown. Finally, two replications accompanied this study to address internal validity. Replications also augment confidence in findings resulting from the study (Horner & Baer, 1978; Horner et al., 2005; Kratochwill et al., 2010).

**Research question one.** Do students with learning disabilities improve their ability to solve word problems with regrouping after receiving an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence? Two sets of data offer insight to answer this question. Presented first are data obtained from the Baseline Probes and Intervention Probes. The second set of data was from the pre- and posttest measures.
Research question two. Do students with learning disabilities improve their computation with regrouping for addition and subtraction after an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence? Scores obtained from pretests and posttests on level of skill mastery and computation fluency reveal an indication of the answer to this question.

Research question three. Do students with learning disabilities report high levels of satisfaction with an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence? To answer this question, satisfaction questionnaires were given to each participant after they finished the posttests. Percent of favorable answers was used to determine overall student satisfaction with the intervention.
Chapter IV

RESULTS

Introduction

The purpose of this research was to study the effects of evidenced-based combined strategy instruction with a graduated learning sequence to teach word problems involving addition and subtraction with regrouping situations to elementary students with learning difficulties. Data were collected to answer three research questions related to the participants: 1) ability to solve word problems using newly taught strategies, 2) ability to increase computation competency in solving addition and subtraction problems involving regrouping, and 3) satisfaction level regarding the intervention lessons. These three areas address the dependent variables of the study. The independent variable is the intervention, designed with staggered, multiple baselines to enhance validity of the results.

This chapter presents findings relative to the three research questions in a sequential fashion. Afterward, interscorer reliability and fidelity of treatment data are presented. The chapter concludes with a summary of the results from this study.
Research Question One

Do students with learning disabilities improve their ability to solve word problems with regrouping after receiving an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence?

Two sets of data were generated to determine participant improvement in word problem-solving involving addition and subtraction with regrouping. The first set of data were created from comparing scores obtained from baseline conditions to those from intervention lessons. The second set of data were comprised of scores earned on pretest compared to posttest.

See Figures 1, 2, and 3 for data collected on word problem-solving during the baseline phase and intervention phase of this study. Visual analysis was used to scrutinize these data. Specifically, level, trend, and variability (Kratochwill et al., 2010) are presented in accordance to the parameters of a multiple probe design.

First, upon visual inspection of these three figures, all seven participants were seen to show improvement in problem solving skill level from baseline to intervention.
Figure 1

Percent Correct Word Problem Responses for Dyad 1
Figure 2

Percent Correct Word Problem Responses from Triad 2
A closer look at baseline probe scores for participants in Dyad 1 shows a mean baseline score of 57.7 with a standard deviation of 11.9, while the mean intervention
score was 94.4 with a standard deviation of 7.0. These values indicate a mean percentage point improvement of 36.6. See Table 3 for baseline and intervention probe percentage scores for participants in Dyad 1.

### Table 3

**Dyad 1 Baseline and Intervention Probe Scores for Word Problem-Solving**

<table>
<thead>
<tr>
<th>Participants</th>
<th>Baseline Probes M/SD</th>
<th>Intervention Probes M/SD</th>
<th>Percentage Point Increase from Baseline to Intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>41.7/8.5</td>
<td>93.8/7.7</td>
<td>52.1</td>
</tr>
<tr>
<td>C-1</td>
<td>63.8/6.0</td>
<td>95.0/6.1</td>
<td>31.2</td>
</tr>
</tbody>
</table>

M: Mean; SD: Standard Deviation

Analyzing baseline probe scores for participants in Triad 2 shows a mean baseline score of 58.8 with a standard deviation of 7.0. The mean intervention score was 96.5 with a standard deviation of 5.2. These values indicate a mean percentage point improvement of 37.3. See Table 4 for baseline and intervention probe percentage scores for participants in Triad 2.
Table 4

Triad 2 Baseline and Intervention Probe Scores for Word Problem-Solving

<table>
<thead>
<tr>
<th>Participants</th>
<th>Baseline Probes M/SD</th>
<th>Intervention Probes M/SD</th>
<th>Percentage Point Increase from Baseline to Intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-2</td>
<td>56.7/8.5</td>
<td>96.3/4.2</td>
<td>39.6</td>
</tr>
<tr>
<td>B-2</td>
<td>62.0/7.7</td>
<td>93.8/4.0</td>
<td>31.8</td>
</tr>
<tr>
<td>C-2</td>
<td>57.5/7.1</td>
<td>97.1/5.9</td>
<td>38.8</td>
</tr>
</tbody>
</table>

Mean; SD: Standard Deviation

Considering baseline probe scores for participants in Dyad 3 reveals a mean baseline score of 65.0 with a standard deviation of 16.6, while the mean intervention score was 97.5 and a standard deviation of 7.5. These values yield a mean percentage point improvement of 32.5. See Table 5 for baseline and intervention probe percentage scores for participants in Dyad 3.
Table 5
Dyad 3 Baseline and Intervention Probe Scores for Word Problem-Solving

<table>
<thead>
<tr>
<th>Participants</th>
<th>Baseline Probes M/SD</th>
<th>Intervention Probes M/SD</th>
<th>Percentage Point Increase from Baseline to Intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-3</td>
<td>83.3/8.5</td>
<td>99.2/2.8</td>
<td>15.9</td>
</tr>
<tr>
<td>B-3</td>
<td>54/8.6</td>
<td>87.5/8.7</td>
<td>33.5</td>
</tr>
</tbody>
</table>

M: Mean; SD: Standard Deviation

The second dimension of visual analysis considered for this study is trend. The trend for the participants showed overall positive slope; however, there were seven instances where lessons were repeated as the scores dipped below the established 80% mastery level. In particular, Lesson 1 for participant A-1, Lessons 3 and 9A for participant A-2, Lesson 11 for participant B-2, Lesson 2 for C-2 and Lessons 7 and 10 for B-3 were repeated before instruction continued. Beyond these occurrences, mastery level of achievement was reached and maintained for all participants and throughout all lessons, yielding a fairly stable trend.

The third and final consideration of visual analysis is variability. The variability in terms of baseline probes and intervention lessons scores for this study can be summarized across participants. Notably, for four participants, the variability from baseline to intervention decreased. For two participants, the variability stayed
virtually the same, and for one subject, variability went up. See Table 6 for a numerical summary of details regarding variability.

Table 6

Summary of Participant Variability in Baseline and Intervention Scores for Word Problem-Solving

<table>
<thead>
<tr>
<th>Group</th>
<th>Participant</th>
<th>B Low</th>
<th>B High</th>
<th>B SD</th>
<th>I Low</th>
<th>I High</th>
<th>I SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dyad 1</td>
<td>A-1</td>
<td>30%</td>
<td>50%</td>
<td>8.5</td>
<td>80%</td>
<td>100%</td>
<td>7.7</td>
</tr>
<tr>
<td>Dyad 1</td>
<td>C-1</td>
<td>55%</td>
<td>75%</td>
<td>6</td>
<td>80%</td>
<td>100%</td>
<td>6.1</td>
</tr>
<tr>
<td>Triad 2</td>
<td>A-2</td>
<td>45%</td>
<td>65%</td>
<td>8.5</td>
<td>90%</td>
<td>100%</td>
<td>4.1</td>
</tr>
<tr>
<td>Triad 2</td>
<td>B-2</td>
<td>55%</td>
<td>65%</td>
<td>4.0</td>
<td>85%</td>
<td>100%</td>
<td>5.3</td>
</tr>
<tr>
<td>Triad 2</td>
<td>C-2</td>
<td>50%</td>
<td>70%</td>
<td>7.1</td>
<td>80%</td>
<td>100%</td>
<td>5.9</td>
</tr>
<tr>
<td>Dyad 3</td>
<td>A-3</td>
<td>75%</td>
<td>95%</td>
<td>8.5</td>
<td>90%</td>
<td>100%</td>
<td>2.8</td>
</tr>
<tr>
<td>Dyad 3</td>
<td>B-3</td>
<td>40%</td>
<td>65%</td>
<td>8.6</td>
<td>80%</td>
<td>100%</td>
<td>8.7</td>
</tr>
</tbody>
</table>

*B=Baseline; **I=Intervention; ***SD=Standard Deviation

Participant improvement for word problem-solving is also evident with the second set of data collected. Pretest and posttest scores reveal gains increase in level across all participants. See Figures 4, 5, and 6 for data collected for word problem-solving during pretest and posttest. The pretest overall range was from 22% to 88% with a mean of 53.1% and standard deviation of 19.1 while that for the posttest range was from 84% to 100% with a mean of 94.6% and standard deviation of 5.7. This
data leads to a mean percentage point improvement of 41.4 from pretest to posttest across participants and gives an overall view of the intervention data trend. Analysis of variability cannot be completed based on only two scores reported (pretest and posttest). See Table 7 for a data summary of pretest and posttest scores.

Figure 4

Dyad 1 Word Problem Scores Pretest (1) to Posttest (2)
Figure 5
Triad 2 Word Problem Scores Pretest (1) to Posttest (2)

Figure 6
Dyad 3 Word Problem Scores Pretest (1) to Posttest (2)
Table 7

Word Problem Pretest and Posttest Data

<table>
<thead>
<tr>
<th>Group</th>
<th>Participant</th>
<th>Pretest Score (%)</th>
<th>Posttest Score (%)</th>
<th>Percentage Point Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dyad 1</td>
<td>A-1</td>
<td>36</td>
<td>88</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>C-1</td>
<td>52</td>
<td>96</td>
<td>44</td>
</tr>
<tr>
<td>Triad 2</td>
<td>A-2</td>
<td>22</td>
<td>96</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>B-2</td>
<td>58</td>
<td>100</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>C-2</td>
<td>60</td>
<td>100</td>
<td>40</td>
</tr>
<tr>
<td>Dyad 3</td>
<td>A-3</td>
<td>88</td>
<td>98</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>B-3</td>
<td>56</td>
<td>84</td>
<td>28</td>
</tr>
<tr>
<td>Mean Values</td>
<td></td>
<td>53.1</td>
<td>94.6</td>
<td>41.4</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td></td>
<td>19.1</td>
<td>5.7</td>
<td>41.4</td>
</tr>
</tbody>
</table>

Research Question Two

Do students with learning disabilities improve their computation with regrouping for addition and subtraction after an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence?

To address the second question for this study, two sets of data were also generated: (1) pretest and posttest scores on computation skills of three-digit addition and subtraction problems involving one or two regrouping situations, and (2) pretest and posttest on computation fluency of three-digit addition and subtraction problems
involving one or two regrouping situations. Figures 7, 8, and 9 display the results of
the computation skills pretest and posttest for Dyad 1, Triad 2, and Dyad 3.

The computation skills pretest range overall was from 45% to 100% with a
mean of 74.3% and standard deviation of 20.6 while the range for posttest was from
75% to 100% with a mean of 95.0% and standard deviation of 9.1. This data leads to
a mean percentage point improvement of 20.7 from pretest to posttest across
participants for computation skills.

Figure 7
Dyad 1 Computation Skills Scores Pretest (1) to Posttest (2)
Figure 8
Triad 2 Computation Skills Scores Pretest (1) to Posttest (2)

Figure 9
Dyad 3 Computation Skills Scores Pretest (1) to Posttest (2)
### Table 8

Computation Skills Pretest and Posttest Data

<table>
<thead>
<tr>
<th>Group</th>
<th>Participant</th>
<th>Pretest Score (%)</th>
<th>Posttest Score (%)</th>
<th>Percentage Point Improvement (Decline)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dyad 1</td>
<td>A-1</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>C-1</td>
<td>85</td>
<td>95</td>
<td>10</td>
</tr>
<tr>
<td>Triad 2</td>
<td>A-2</td>
<td>50</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>B-2</td>
<td>60</td>
<td>100</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>C-2</td>
<td>45</td>
<td>100</td>
<td>55</td>
</tr>
<tr>
<td>Dyad 3</td>
<td>A-3</td>
<td>85</td>
<td>75</td>
<td>(10)</td>
</tr>
<tr>
<td></td>
<td>B-3</td>
<td>95</td>
<td>95</td>
<td>0</td>
</tr>
<tr>
<td>Mean Values</td>
<td></td>
<td>74.3</td>
<td>95.0</td>
<td>20.7</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td></td>
<td>20.6</td>
<td>9.1</td>
<td></td>
</tr>
</tbody>
</table>

The second set of data collected addressing computation skills also reveals improvement. These data were gathered from pretest and posttest scores of computation fluency, reported as correct digits per minute (cdpm), of three-digit addition and subtraction problems including one or two regrouping situations. See Figures 10, 11, and 12 for pretest and posttest scores on computation fluency for each group of participants.
Figure 10
Dyad 1 Computation Fluency Scores Pretest (1) to Posttest (2)

Figure 11
Triad 2 Computation Fluency Scores Pretest (1) to Posttest (2)
The computation fluency pretest range overall was from 1 cdpm to 10 cdpm with a mean of 6.2 and standard deviation of 2.6. Scores on the posttest show an increased level of performance for each participant, and ranged from 8.5 cdpm to 27 cdpm with a mean of 13.8 cdpm and standard deviation of 5.8. Additionally, the percent improvement of correct digits per minute for computation fluency ranged from 15% improvement to 750% improvement. This data lead to a mean percentage point improvement of 121.8 from pretest to posttest across participants and gives a snapshot of the overall improvement across participants. See Table 9 for a data summary of pretest and posttest scores for computation fluency.
Table 9

Computation Fluency Pretest and Posttest Data

<table>
<thead>
<tr>
<th>Group</th>
<th>Participant</th>
<th>Pretest Score (cdpm*)</th>
<th>Posttest Score (cdpm)</th>
<th>Improvement (cdpm)</th>
<th>Percent Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dyad 1</td>
<td>A-1</td>
<td>10</td>
<td>11.5</td>
<td>1.5</td>
<td>15.0%</td>
</tr>
<tr>
<td></td>
<td>C-1</td>
<td>7</td>
<td>27</td>
<td>20</td>
<td>285.7%</td>
</tr>
<tr>
<td>Triad 2</td>
<td>A-2</td>
<td>1</td>
<td>8.5</td>
<td>7.5</td>
<td>750.0%</td>
</tr>
<tr>
<td></td>
<td>B-2</td>
<td>8</td>
<td>10</td>
<td>2</td>
<td>25.0%</td>
</tr>
<tr>
<td></td>
<td>C-2</td>
<td>5.5</td>
<td>13</td>
<td>7.5</td>
<td>136.4%</td>
</tr>
<tr>
<td>Dyad 3</td>
<td>A-3</td>
<td>7</td>
<td>16</td>
<td>9</td>
<td>128.6%</td>
</tr>
<tr>
<td></td>
<td>B-3</td>
<td>5</td>
<td>10.5</td>
<td>5.5</td>
<td>110.0%</td>
</tr>
<tr>
<td>Mean Values</td>
<td></td>
<td>6.2</td>
<td>13.8</td>
<td>7.6</td>
<td>121.8%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td></td>
<td>2.6</td>
<td>5.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

cdpm: Correct digits per minute

Research Question Three

Do students with learning disabilities report high levels of satisfaction with an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence?

After completing the final lesson of the intervention and finishing the three posttests, students were given a series of questions to answer regarding how well they liked certain parts of the intervention. Table 10 summarizes the responses to each question across participants. Note some questions were worded such that a Frown
(No) selection represents a favorable response. Thus, percentage of empiric answers given by the participants are reported as well as answers equating to positive (favorable) and negative (unfavorable) responses in terms of opinions regarding the study.

Table 10

Intervention Satisfaction Questionnaire Summary

<table>
<thead>
<tr>
<th>Question</th>
<th>Frown (No)</th>
<th>Smile (Yes)</th>
<th>Percent Unfavorable Response</th>
<th>Percent Favorable Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Learning how to solve word problems was easy for me.</td>
<td>0</td>
<td>7</td>
<td>0.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>2. I liked working on the learning sheets.</td>
<td>1</td>
<td>6</td>
<td>14.3%</td>
<td>85.7%</td>
</tr>
<tr>
<td>3. Learning how to solve word problems was hard for me</td>
<td>3</td>
<td>4</td>
<td>57.1%</td>
<td>42.9%</td>
</tr>
<tr>
<td>4. The learning sheets were hard for me</td>
<td>5</td>
<td>2</td>
<td>28.6%</td>
<td>71.4%</td>
</tr>
<tr>
<td>5. I liked the learning contract.</td>
<td>0</td>
<td>7</td>
<td>0.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>6. I know more about math now.</td>
<td>1</td>
<td>6</td>
<td>14.3%</td>
<td>85.7%</td>
</tr>
<tr>
<td>7. I am better at adding and subtracting with regrouping now.</td>
<td>0</td>
<td>7</td>
<td>0.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>8. I will use FAST RENAME when I solve word problems.</td>
<td>1</td>
<td>6</td>
<td>14.3%</td>
<td>85.7%</td>
</tr>
<tr>
<td>9. I liked looking at the progress chart to see the results of my work.</td>
<td>0</td>
<td>7</td>
<td>0.0%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>
The mean positive response from the questionnaire was 6.4, yielding 91% positive responses to the questions regarding the intervention study, while the negative responses yielded a mean of 0.6, or 8.6% responses.

Interscorer Reliability

Interscorer reliability was assessed by comparing score computations on at least 20% of the collected data from an outside consultant to the study. Table 11 reflects the interscorer agreements calculated as number of agreements/number of agreements + number of disagreements X 100. The measures thus compared were in the areas of pretests and posttest—word problem, computation skills, and computation skills fluency—and also across probes. The range of percentage agreement was from 93.8% to 100%, yielding a mean percent agreement of 97.3.
Table 11

Interscorer Reliability

<table>
<thead>
<tr>
<th>MEASURE</th>
<th>TOTAL AGREEMENT</th>
<th>TOTAL AGREEMENTS + DISAGREEMENTS</th>
<th>PERCENTAGE OF AGREEMENT (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Word Problem Pretest</td>
<td>96</td>
<td>100</td>
<td>96.0</td>
</tr>
<tr>
<td>Computation Skills Pretest</td>
<td>40</td>
<td>40</td>
<td>100.0</td>
</tr>
<tr>
<td>Computation Fluency Pretest</td>
<td>26</td>
<td>26</td>
<td>100.0</td>
</tr>
<tr>
<td>Word Problem Probes</td>
<td>352</td>
<td>360</td>
<td>97.8</td>
</tr>
<tr>
<td>Word Problem Posttest</td>
<td>96</td>
<td>100</td>
<td>96.0</td>
</tr>
<tr>
<td>Computation Skills Posttest</td>
<td>39</td>
<td>40</td>
<td>97.5</td>
</tr>
<tr>
<td>Computation Fluency Posttest</td>
<td>30</td>
<td>32</td>
<td>93.8</td>
</tr>
<tr>
<td>Total</td>
<td>679</td>
<td>698</td>
<td>97.3</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td>97.3</td>
</tr>
</tbody>
</table>

Fidelity of Treatment

Thirteen sessions across the three groups (31%) were observed to determine fidelity of treatment. Specifically, four sessions of Group A, two sessions of Group B, and seven sessions of Group C were observed. Observing the sessions in this manner ensured the groups received the same content and instructional procedures, which enhances reliability of the study procedures and outcomes. The *Fidelity of Treatment Checklist* was developed for this purpose (Appendix Q). The components on the
checklist include: (1) advance organizer, (2) describe and model, (3) guided practice, (4) independent practice, (5) computation practice, and (6) score and provide feedback. The observer reported all components of explicit instruction (100%) for this study were implemented during the sessions.

Summary

Results of this study gave insight to the effect of the combination of explicit instruction and carefully selected cognitive strategies, along with a graduated lesson sequence of word problem lessons on students with math difficulties. Also revealed were the results of computation skills of the participants for solving addition and subtraction with regrouping problems from pretest to posttest. Moreover, the satisfaction with receiving intervention lessons used in this study was summarized. Chapter V provides a discussion of these results.
Chapter V

DISCUSSION, RECOMMENDATIONS, AND CONCLUSIONS

Introduction

Included in this chapter is a summary of the findings for this study. Also, links to current research, observations, limiting factors, and implications for the future are addressed. Research in the area of solving mathematical word problems is limited compared to that which is available for basic math fact solving. Research with word problems involving the express operation of addition or subtraction requiring regrouping is limited as well.

In an effort to extend existing research in the area of word problem-solving involving addition and subtraction with regrouping, the following questions have been addressed: (1) Do students with learning disabilities improve their ability to solve word problems with regrouping after receiving an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence? (2) Do students with learning disabilities improve their computation with regrouping for addition and subtraction after an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence? (3) Do students with learning disabilities report
high levels of satisfaction with an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence?

Chapter V is organized by discussing each research question in a sequential format, followed by a discussion of inter-rater reliability and fidelity of treatment. The chapter will end with detailing conclusions based on the results of the study, and making recommendations for future research.

DISCUSSION

Research Question One

Do students with learning disabilities improve their ability to solve word problems with regrouping after receiving an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence? Two sets of data were collected to address this question: Continuous monitoring of performance on probes during the baseline and intervention phases and scores obtained from pretest and posttest for the intervention. First an analysis of three components of visual inspection (level, trend, and variability) will be given with regard to the baseline and intervention data.

Although there was variation between subjects with respect to mean baseline scores—41.2 to 83.3—the level of performance increased for each of the seven participants between baseline and intervention. For example, the mean baseline score for participant A-1 was 41.2%; however, the mean score during intervention was 93.8%, resulting in an increased performance level of 52.6 percentage points.
Although not quite as dramatic, all remaining participants enjoyed increased performance levels between baseline and intervention. This indication supports the conclusion that the intervention was successful. However, trend and variability of the data must also be addressed.

The second benchmark for visual analysis is trend. This characteristic of data is best defined as the slope of the best-fit line within a phase. Baseline trend shows a flat or negative slope for all subjects except A-2. Here, the baseline trend has a positive slope, which could be indicative of a learning effect. However, Group A participants were only afforded three data points to establish baseline due to the interest of time. Considering the significant increase in level for A-2 between baseline and intervention helps substantiate the overall learning effect between baseline and intervention. Still, due to the positive trend in baseline, the argument for successful intervention learning is somewhat weakened for A-2.

All the trend lines for the participants during intervention phase were positive except for A-3 and B-3. Upon closer inspection of the data, the intervention scores for A-3 were mostly at 100%, with only one score of 90%. This may reveal a limitation to using trend as an indicator of effect. There is a limit to how well scores can reflect learned content when represented by percentages. The ceiling is 100% and cannot go any higher. Therefore, the highest achievers will display a flat learning line—not indicative of their capability at all. However, when coupled with the positive change in level, a learning effect is demonstrated.
Another negative trend, however slight, is noted for B-3. Even so, the scores for intervention of B-3 were all at mastery level or above (80%). The positive change in level from baseline to intervention is considerably more dramatic than for A-3 (33.5). This, coupled with the fact that all intervention scores are in the mastery range, must be considered to determine an overall successful learning effect for intervention.

The third criterion for visual inspection is variability and can be represented as the standard deviation from the mean in both baseline and intervention phases. Four of the seven participants produced standard deviations that decreased from baseline to intervention. For two participants, the standard deviation remained virtually the same. The standard deviation for the remaining subject went up from baseline to intervention. In classical behavior analysis, this tendency might create hesitation to deem the study results successful. However, the essence of the intervention must be dissected.

In familiar AB design, the intention of the intervention is a behavior to be learned as a result of a distinct, static event—a stimulus. In contrast, the stimulus for the treatment (intervention) phase of this study was not static. One of the design parameters built into the intervention was a graduated lesson sequence. That is, more complex material was encountered with each successive lesson. Therefore, the dips displayed during the intervention phase in Figures 1, 2, and 3, reflect continuous adjustment to learning new and more complicated material. In addition, the entirety of the scores graphed in those figures is at or above the mastery level of 80%.
Consequently, the variation parameter of visual analysis does not carry as much weight in determining if the intervention produced a successful learning effect.

Overall, visual analysis in terms of level, trend, and variability was completed. All participants improved their level of problem solving competencies. The percentage point improvements from baseline to intervention were higher than 31 for the participants. Mean intervention scores, with the exception of one, were 91% or better. Most of the participant trends showed positive slopes. Finally, the variation in terms of standard deviations was shown to increase for some, decrease for some, and for two, the standard deviation did not substantially increase or decrease. These findings will be compared to the second set of data obtained to indicate the success of learning effect.

Scores on pretest and posttest for word problem-solving were presented in Chapter IV in Figures 4, 5, and 6. Trends for each participant clearly reveal positive slopes from pretest to posttest with percentage improvements ranging from 11% for A-3 to 336% for A-2. Notably, the range of scores for A-3 was from 88% to 98%, while that for A-2 was 22% to 96%. Analysis of visual criteria for level and variability are not applicable for this data since more than one data point is required to determine both level and variability for pretest and posttest situations.

Therefore, considering visual analysis of baseline to intervention condition, as well as data from pretest to posttest, it is reasonable to conclude convergence of data indicates a positive, successful learning effect due to intervention on word problem solving for all participants. This conclusion is consistent with success of using
explicit instruction coupled with cognitive learning strategies shown by Swanson and Hoskyn (1998).

Findings for this research question is also supported by the success of adding mnemonics and a graduated lesson sequence to explicit instruction for solving word problems (Ferreira, 2009; Flores et al., 2014; Kaffar, 2014b; Mancl, 2011; Miller & Kaffar, 2011a; Miller & Kaffar, 2011b; Miller & Mercer, 1993a; Miller & Mercer, 1993b). It is also in accordance with results using explicit instruction and cognitive learning strategies in the form of SBI for solving word problems (Beckmann, 2004; England, 2010; Foster, 2007; Gersten et al., 2009; Griffin & Jitendra, 2008; Jitendra & Hoff, 1996; Jitendra et al., 2007a; Schwab et al., 2013; van Garderen et al., 2012). In addition, it is consistent with findings that support modifying linguistics for successful word problem solving with elementary students exhibiting math difficulties (Abedi & Lord, 2001; Fuchs et al., 2006; Jitendra et al., 2013b; Reusser, 2000; Vilenius Tuohimaa et al., 2008). Moreover, creating meaningful context, relating to participants’ experiential background, and developing a relaxed, low-stress environment for learning might well have aided WM to free up (Goodwin, 2014; Zheng et al., 2011). This, in turn would allow more working memory capacity for focusing on learning during intervention.

**Research Question Two**

Do students with learning disabilities improve their computation with regrouping for addition and subtraction after an intervention that involves explicit
instruction, cognitive strategies, and a graduated lesson sequence? Two sets of data were collected to answer this question. First, scores from pretest to posttest measuring computation ability on twenty numerical problems were obtained. Results from this untimed assessment show that for all participants except one, posttest scores went up or stayed the same.

Two of the participants’ scores on posttest did not increase from pretest. It is important to point out that for A-1, 100% was achieved in both situations, and for B-3, 95% was the score for pretest and posttest. A 95% score translates to missing one problem on the test.

The pretest result for A-3 was 85% (a result from missing three problems) while that for the posttest was 75% (a result of missing two more problems than on pretest). The decline overall for A-3 is 12%. Being mindful of the performance on intervention probes for A-3 is important, along with the fact that word problem solving involves computation skills as only part of the strategy leading to success. Therefore the decline A-3 evidenced on computation posttest conceivably could be due to an unknown, outside factor. Notwithstanding, this data will be considered concurrently with the next set of data used to answer research question two.

It is interesting to note the most dramatic increase in computation scores was from C-2, increasing from 45% correct on pretest to 100% correct on posttest.

The second set of data used to affirm success in computation skills was from pretest and posttest scores on computation fluency. These tests were timed for 2 minutes and results were reported as correct digits per minute. Figures 10, 11, 12
graphically display these results. Results indicate that all seven subjects experienced positive growth in fluency outcomes from pretest to posttest. A-1 increased the pretest result of 10 cdpm to 11.5 cdpm, yielding a 1.3 cdpm increase and a 15% increase overall. C-1 improved fluency outcomes from pretest to posttest by 20 cdpm, or by 285.7%. In contrast, A-2 went from 1 cdpm on pretest to 8.5 cdpm on posttest which is an increase of 750%.

Indications from pretest to posttest for computation fluency lead to a positive learning effect of computation skills. This information, coupled with that from the computation skills pretest and posttest, indicate a positive learning effect due to intervention for overall computation ability. These findings are consistent with others who found increased computation skills as a result of learning a related skill and embedded repeated practice (Carmack, 2011; Ferreira, 2009; Zheng et al., 2012).

Research Question Three

Do students with learning disabilities report high levels of satisfaction with an intervention that involves explicit instruction, cognitive strategies, and a graduated lesson sequence? To answer this question students responded to 10 yes and no questions after intervention lessons and posttests were complete. The mean positive response from the questionnaire was 6.4, yielding 91% positive responses to the questions regarding the intervention study, while the negative responses yielded a mean of 0.6, or 8.6% responses. Overall, these figures provide a glimpse of positive
feelings regarding the intervention which is consistent with other researchers using similar tactics (Carmack, 2011; Ferreira, 2009; Flores et al., 2014; Mancl, 2011)

Particularly revealing was half of the questions received 100% agreement among the respondents. Those items were: #1 Learning how to solve word problems was easy for me; #5 I liked the learning contract; #7 I am better at adding and subtracting with regrouping now; #9 I liked looking at the progress chart to see the results of my work; and #10 I think other students should learn to solve word problems this way. These survey results suggest the participants had a high level of satisfaction with the intervention.

**Interscorer reliability.** Interscorer reliability was determined by having an outside consultant to the research rescore 698 data evaluations. This equates to 20% or more of the data accrued during this study. The range in agreement between these scores and those evaluated by the principal investigator is 93.8% to 100%. These values are well within the range for minimum acceptable values of 80% to 90% as set forth by Hartmann, Barrios, and Wood (2004) (as cited in Kratochwill et al., 2010).

**Fidelity of treatment.** The supervising professor for the principal investigator observed 31% of the sessions; which included random representations of all participants. Results show 100% of the criteria were adhered to during teaching these sessions. This suggests high fidelity of treatment for the implementation of the intervention across all participants.
Recommendations

First, it is difficult with many studies involving special education to include a sample size large enough to lend strong credence to statistical analysis. Notwithstanding, it can be done. Not only replicating this study would substantiate its findings, but also doing so with a larger sample size would support stronger statistical analysis of the data.

A second recommendation to ensure reliability of results would be to shore up fidelity by having scripted lessons in place. Especially if more than one instructor gets involved with future studies. By providing scripted lessons and written guidance in how to carry out instruction, consistency over implementation within and across instructor lessons can be increased.

Finally, the window of time to conduct lessons for this study was 30 minutes. Although the instructional window should not be too big (Griffin & Jitendra, 2008), allowing 50 minutes would generate enough time to complete individual lessons for most students within one lesson window of time. The shorter, 30-minute lessons ended up lengthening the number of days spent on the intervention since participants could not finish their learning sheets by the end of a one-day lesson. This can interfere with teaching other IEP-related math curricula.

Conclusion

In conclusion, results gathered from this study meet evidence standards for determining a learning effect as a result of intervention as outlined in Kratochwill
et al. (2010). Specifically: (a) the PI determined how to systematically manipulate the independent variable, (b) minimum thresholds were met from inter-scorer reliability checks, (c) at least three attempts to determine intervention effect with three phase repetitions were included, and (d) there were at least three data points defining each phase of the study.

As researchers develop ways for students to succeed in mathematics, these same students stand a significant advantage for leading successful and happy lives. Indeed, by increasing the ability for solving word problems, students may also increase opportunities to story their lives with a higher degree of satisfaction.
REFERENCES


APPENDICES
APPENDIX A

Parental Informed Consent
Investigating the Effects of Evidence-Based Strategies on Word Problems With Regrouping for Elementary Students

Parent/Guardian Consent Form

My name is Margaret Vanderwarn, and I am a special education teacher at Mississippi Heights Elementary School in the Sauk Rapids-Rice Public Schools. I am also a graduate student at St. Cloud State University. This form is being sent to ask your permission to allow your child to participate in a study being conducted for my Master’s Degree at St. Cloud State University. Two consent forms—one for you, the parent/guardian, and the other for your child—are included with this memo. If you and your child would like to participate, both of these forms must be signed and returned. If your child is unable to read the student consent form, please take a few moments to read it to him/her and explain it as needed.

Background Information and Purpose
It is my intent to use evidence-based strategies to enhance the instruction of word problems with regrouping in math. Collecting this data will assist in the instruction of future teachers and their students who struggle in math.

Procedures
Each student will receive pretests and posttests to determine their improvement after receiving instruction. Each student will receive approximately twelve lessons, and Progress Charts will be used to document improvement. Additionally, your child will be asked to compute similar problems after the intervention lessons have completed to determine if they maintained the newly taught skills. At the time you received this letter, some data has already been collected, while the last remaining data points remain to be completed. We ask permission to use this already existing information and any other which may be collected throughout the remaining weeks of school.

To better compare the data from this study and that of other studies, some information regarding your child’s demographics and academic history will be obtained from his or her cumulative school record folder. Specifically, disability category, age, grade, gender, and ethnicity will be collected as well as your child’s standard score from the most recently administered ability assessment. This information will be compiled and used with complete anonymity and confidentiality, as your child will be given a coded identifier to be used throughout the research process (a letter-number combination).

In addition, this project will in no way reflect poorly on your child’s grade report. It is the hope that these lessons will make learning how to solve word problems with regrouping more concrete for each student.

Risks
There are no foreseeable risks associated with participation in this study.

Benefits
Data will be collected during the study and will include still pictures taken throughout the study. These pictures will not include your child, but rather, the photos will be of the worksheets and graphs created by your child. No identifying information of your child will be exposed in this process. It is anticipated that the data and photos collected will be used for educational purposes only, such as seminars, conventions, courses at St. Cloud State University, and possibly publication in an academic journal.

Confidentiality
In addition to using data for the final paper that will remain on permanent file at the St. Cloud State University Miller Learning Resources Center (library), data may also be published in professional
journals at a later time. At no time during the study or when reporting the findings will your child's name be used in any manner.

Research Results
The data collected will be used to further our knowledge of effective ways to teach word problems with regrouping. Also, the results of the study will help to prepare future special education teachers. At your request, I am happy to provide a summary of the research results when the study is completed.

Contact Information
If you have questions or concerns involving this study, you may contact me via email at yama1203@stcloudstate.edu. You may also contact my supervisor, Dr. Bradley J. Kaffar, at bkaffar@stcloudstate.edu or (320) 308-3267.

Voluntary Participation/Withdrawal
Participation in the study is completely voluntary, and your child can withdraw at any time without any penalty or harm to him/her in regards to passing his/her math class. Children desiring not to participate in the study will continue their math instruction with another class at the same instructional level as taught in the study. Throughout the study, your child will be asked how they are feeling and if they want to continue or withdraw from the study. Their responses will be recorded. In addition, my supervisor from St. Cloud State University, Dr. Bradley J. Kaffar, will visit the school, observe how the study is progressing, and make suggestions as needed. If your child decides to participate, he/she is free to withdraw at any time without penalty.

Acceptance to Participate
Your signature indicates that you and your child have read the information provided here and have decided to participate. You or your child may withdraw from the study at any time without penalty after signing this form.

I look forward to having your children participate in this innovative study, and I thank you in advance for your cooperation as I continue to complete my Master's Degree at St. Cloud State University.

Consent Form for Math Instruction Study.
Please return to Ms. Vanderwarr as soon as possible.

- I grant permission to have my child participate in the master's study conducted by Ms. Margaret Vanderwarr.
- I understand that the study involves pretests, posttests, maintenance skills tests, and approximately twelve lessons.
- I give my permission to have visual (still pictures) recordings made of the artifacts my child created as a participant in this study, and that there will be no identifiable image or likeness of my child in the photos.
- I realize that data will be collected and may be used at educational conferences/seminars, including data regarding demographics and score reports. This information will be identified with a code rather than my child's name to ensure anonymity.
- I realize that the results of the study, including visual photos of the artifacts my child created as a result of this study, may be used in professional publications at a later date. These photos will include no identifiable information or likeness of my child.
- I further grant permission for Ms. Vanderwarr to obtain demographic information of my child's disability category, grade, age, gender, and ethnicity as well as the standard score of the most recently administered ability assessment of my child.

St. Cloud State University
Institutional Review Board
Approval date: 4-24-15
Expiration date: 4-23-16
• I understand that confidentiality will be maintained and that my child's name will not be used in any manner while conducting the study or reporting the results of the study.
• I further understand that my child may withdraw from the study at any time if he/she so desires without any harm in regard to his/her educational progress.

X______________________________
Student Name (Printed)

X______________________________
Parent/Guardian Name (Printed)

X______________________________
Parent/Guardian Signature

St. Cloud State University
Institutional Review Board
Approval date: 4-24-15
Expiration date: 4-23-16
APPENDIX B

Child Assent
Better Ways to Solve Word Problems!

Child Assent

Hi there, __________________!

You know I love teaching math, right? And I would love teaching you some cool ways to help you learn how to solve word problems!

Even if you are not that happy with learning math, I'd be willing to bet you'd really feel good about being able to correctly solve three-digit adding and subtracting where you need to regroup. Even more, you will learn some tricks to help you get everything you need from a word problem paragraph, then set up the right equation, and finally solve it correctly.

The secret is learning some steps you can always use, over and over again, to help you become a successful word problem solver! There would be some pre-tests, about 12 lessons, and some posttests. The fun part is watching to see if your scores go up and up after the pre-test and after lessons have begun. We will also check to see if you will remember these steps after the lessons are over.

The lessons will be in my (Ms. Vanderwarn's) room during your math "What I Need" (WIN) time. This will take about seven weeks to finish. Would you like to join us?

If during this study you decide that you do not want to be a part of the study after all, you need to tell Ms. Vanderwarn or your parents. Your decision to be in the study or not is yours to make and either way is just fine.

When you sign your name on the line with the "X" it means you understand this information and have agreed to be a part of the study. If you do not like being in the study at any time, you may tell Ms. Vanderwarn and she will see that you are put in a different class.

I can't wait to see how your scores improve! Can you?

______________________________
Name (Printed)

______________________________
(Signature)

______________________________
Date

St. Cloud State University
Institutional Review Board
Approval date: 4-24-15
Expiration date: 4-23-16
APPENDIX C

Addition and Subtraction with Regrouping Word Problem Pretest
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. There are 258 boys and 168 girls at the school. There are 321 math books. How many children are at the school?</td>
<td></td>
</tr>
<tr>
<td>2. Mom has 314 points. Joe has 127 points. Mom has how many more points than Joe?</td>
<td></td>
</tr>
<tr>
<td>3. Sam had 183 coins. Mike gave him 119 more coins. How many coins does Sam have altogether?</td>
<td></td>
</tr>
<tr>
<td>5. Pat had 374 baseball cards. He sold 188 baseball cards. How many baseball cards does he have left?</td>
<td></td>
</tr>
</tbody>
</table>
6. There are 105 red markers and 217 green markers in the box. How many markers are in the box?

7. Lucy planted 177 seeds. Bill planted 346 seeds. How many fewer seeds did Lucy plant than Bill?

8. We saw 277 cars parked in the lot. Then, 144 more cars parked there. We saw 168 bikes. How many cars are in the lot now?

9. Ben ran 205 races. Dad ran 167 races. How many more races did Ben run than Dad?

10. Kate had 342 cans and 417 straws. She sold 168 cans. How many cans does Kate have now?
APPENDIX D

Word Problem Scoring Rubric
## Word Problem Scoring Rubric

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Description</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>F Correct label for answer</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>A Correct number identification for solving</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>S Correct equation set-up</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>T Correct operation identification</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>RENAME Correct computation</td>
<td>0</td>
</tr>
</tbody>
</table>

Total Points: 5
APPENDIX E

Addition and Subtraction with Regrouping Computation Pretest
<p>| | | | | |</p>
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<tr>
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<td>342</td>
<td>205</td>
<td>106</td>
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<td></td>
</tr>
<tr>
<td>483</td>
<td>314</td>
<td>358</td>
<td>944</td>
<td></td>
</tr>
<tr>
<td>+119</td>
<td>-127</td>
<td>+168</td>
<td>-572</td>
<td></td>
</tr>
<tr>
<td>342</td>
<td>144</td>
<td>105</td>
<td>204</td>
<td></td>
</tr>
<tr>
<td>-118</td>
<td>+277</td>
<td>+217</td>
<td>-121</td>
<td></td>
</tr>
<tr>
<td>355</td>
<td>483</td>
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<td>264</td>
<td></td>
</tr>
<tr>
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<td>+572</td>
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</tr>
<tr>
<td>713</td>
<td>268</td>
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</tbody>
</table>
APPENDIX F

Addition and Subtraction with Regrouping Computation Fluency Pretest
### Regrouping Fluency

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
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<td>342</td>
<td>144</td>
<td>105</td>
<td>204</td>
</tr>
<tr>
<td>- 118</td>
<td>+ 277</td>
<td>+ 217</td>
<td>- 121</td>
</tr>
<tr>
<td>355</td>
<td>483</td>
<td>358</td>
<td>264</td>
</tr>
<tr>
<td>+ 188</td>
<td>- 149</td>
<td>- 167</td>
<td>+ 572</td>
</tr>
<tr>
<td>713</td>
<td>268</td>
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<td>+ 371</td>
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<td>342</td>
<td>205</td>
<td>106</td>
</tr>
<tr>
<td>- 127</td>
<td>+ 464</td>
<td>- 163</td>
<td>+ 225</td>
</tr>
</tbody>
</table>
APPENDIX G

Word Problems with Regrouping Progress Chart
Word Problems With Regrouping Progress Chart

Problem-Solving Graph

Computation Graph
APPENDIX H

Learning Contract
Learning Contract

I, __________________________, agree to learn *Word Problems With Regrouping*. If I work hard, I will learn to solve word problems that require regrouping quickly and accurately. This will help me understand math and get better grades.

_________________________  __________
Student Signature          Date

I, __________________________, agree to do whatever I can to help you learn to solve word problems that require regrouping. I will follow the instructions outlined in the book, *Word Problems With Regrouping*, and I will put creative energy into showing you how problem solving is a valuable skill to learn.

_________________________  __________
Teacher Signature          Date
APPENDIX I

Word Problem Probe
1. Matt caught 123 baseballs and ran to 118 bases. Joe caught 177 baseballs. How many baseballs were caught?

2. Sam rode 346 miles on a bike. Bill rode 157 miles on a bike. Sam rode how many more miles than Bill?

3. Mom put 105 dimes in a cup. Dad put 197 dimes in the cup. How many dimes are in the cup now?

4. Ella had 300 pieces of candy. She gave 125 pieces of candy to her sister. How many pieces of candy does Ella have left?
APPENDIX J

Learning Sheet
## Describe and Model

1. Mom put 286 dimes in a cup. Dad put 141 dimes in the cup. How many dimes are in the cup now?

2. Anna had 138 cans in a truck. She put 127 more cans in the truck. How many cans are in the truck now?

## Guided Practice

3. Eric had 124 beans in a jar. He put 193 more beans in the jar. How many beans are in the jar now?

4. Sam had 239 coins. Mike gave him 117 more coins. How many coins does Sam have altogether?
**Independent Practice**

5. We saw 126 cars parked in the lot. Then, 229 more cars parked there. How many cars are in the lot now?

6. Bill had 339 baseball cards. He got 112 more. How many baseball cards does Bill have altogether?

7. Matt planted 172 seeds in the dirt. Abby planted 245 seeds in the dirt. How many seeds are in the dirt now?

**Computation Practice**

8) \[245 + 372\]
9) \[336 - 153\]
10) \[143 + 192\]
11) \[445 - 282\]
12) \[135 + 216\]
APPENDIX K

Satisfaction Questionnaire
INVESTIGATING THE EFFECTS OF EVIDENCE-BASED STRATEGIES ON WORD PROBLEMS WITH REGROUPING

Subject Questionnaire

1. Learning how to solve word problems was easy for me.

2. I liked working on the learning sheets.

3. Learning how to solve word problems was hard for me.

4. The learning sheets were hard for me.

5. I liked the learning contract.

6. I know more about math now.

7. I am better at adding and subtracting with regrouping now.

8. I will use FAST RENAME when I solve word problems.

9. I liked looking at the progress chart to see the results of my work.

10. I think other students should learn to solve word problems this way.
APPENDIX L

RENAME for Addition Reference Card
Addition With Regrouping

Step 1: **Read the problem.**

Step 2: **Examine the ones column: 10 or more, go next door.**

Step 3: **Note ones in the ones column.**

Step 4: **Address the tens column: 10 or more, go next door.**

Step 5: **Mark tens in tens column.**

Step 6: **Examine and note hundreds; exit with a quick check.**

“10 or More” Sentences

Adding the Ones:
*If adding the numbers in the ones column results in 10 or more, regroup to form a ten (10 or more, go next door).*

Adding the Tens:
*If adding the numbers in the tens column results in 10 or more, regroup to form a hundred (10 or more, go next door).*
APPENDIX M

RENAME for Subtraction Reference Card
Subtraction With Regrouping

Step 1: Read the problem.

Step 2: Examine the ones column: Use the BBB Sentence for ones.

Step 3: Note ones in the ones column.

Step 4: Address the tens column: Use the BBB Sentence for tens.

Step 5: Mark tens in the tens column.

Step 6: Examine and note hundreds; exit with a quick check.

BBB Sentences

BBB Sentence for Ones:
*Bigger number on bottom means break down a ten and trade.*

BBB Sentence for Tens:
*Bigger number on bottom means break down a hundred and trade.*
APPENDIX N

FAST RENAME Reference Card
The “FAST RENAME” Strategy

Step 1: **Find what you’re solving for.**

Step 2: **Ask yourself, “What are the parts of the problem?”**

Step 3: **Set up the numbers.**

Step 4: **Tie down the sign.**

<table>
<thead>
<tr>
<th>Addition With Regrouping</th>
<th>Subtraction With Regrouping</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1:</strong> Read the problem.</td>
<td><strong>Step 1:</strong> Read the problem.</td>
</tr>
<tr>
<td><strong>Step 2:</strong> Examine the ones column: 10 or more, go next door.</td>
<td><strong>Step 2:</strong> Examine the ones column: use the BBB Sentence for ones.</td>
</tr>
<tr>
<td><strong>Step 3:</strong> Note ones in the ones column.</td>
<td><strong>Step 3:</strong> Note ones in the ones column.</td>
</tr>
<tr>
<td><strong>Step 4:</strong> Address the tens column: 10 or more, go next door.</td>
<td><strong>Step 4:</strong> Address the tens column: use the BBB Sentence for tens.</td>
</tr>
<tr>
<td><strong>Step 5:</strong> Mark tens in the tens column.</td>
<td><strong>Step 5:</strong> Mark tens in the tens column.</td>
</tr>
<tr>
<td><strong>Step 6:</strong> Examine and note hundreds; exit with a quick check.</td>
<td><strong>Step 6:</strong> Examine and note hundreds; exit with a quick check.</td>
</tr>
</tbody>
</table>

**“10 or More” Sentences**

**Adding the Ones:**
If adding the numbers in the ones column results in 10 or more, regroup to form a ten (10 or more, go next door).

**Adding the Tens:**
If adding the numbers in the tens column results in 10 or more, regroup to form a hundred (10 or more, go next door).

**BBB Sentences**

**BBB Sentence for Ones:**
Bigger number on Bottom means Break down a ten and trade.

**BBB Sentence for Tens:**
Bigger number on Bottom means Break down a hundred and trade.
APPENDIX O

Extraneous Material Reference Card
extraneous

extra

irrelevant

nonessential

Word problems may contain extraneous information.
APPENDIX P

Graduated Lesson Sequence
Graduated Lesson Sequence

Pretests
Lesson 1A: Introduce the “RENAME” Strategy for Three-Digit Addition With Regrouping From Ones to Tens or Tens to Hundreds
Lesson 1B: Introduce the “RENAME” Strategy for Three-Digit Subtraction With Regrouping From Tens to Ones or Hundreds to Tens
Lesson 2: Introduce the “FAST RENAME” Strategy for Word Problems With Regrouping
   - Addition With Regrouping From Ones to Tens or Tens to Hundreds
   - Subtraction With Regrouping From Tens to Ones or Hundreds to Tens
Lesson 3: Introduce the “Add To” Situation (Result Unknown): Apply the “FAST RENAME” Strategy to Word Problems With Regrouping
   - Addition With Regrouping From Ones to Tens or Tens to Hundreds
   - Subtraction With Regrouping From Tens to Ones or Hundreds to Tens
Lesson 4: Introduce the “Take From” Situation (Result Unknown): Apply the “FAST RENAME” Strategy to Word Problems With Regrouping
   - Addition With Regrouping From Ones to Tens or Tens to Hundreds
   - Subtraction With Regrouping From Tens to Ones or Hundreds to Tens
Lesson 5: Begin Generalization of “Add To” and “Take From” Situations: Apply the “FAST RENAME” Strategy to Word Problems With Regrouping
   - Addition With Regrouping From Ones to Tens or Tens to Hundreds
   - Subtraction With Regrouping From Tens to Ones or Hundreds to Tens
Lesson 6: Introduce Word Problems Containing Extraneous Information and Continue Generalization of “Add To” and “Take From” Situations: Apply the “FAST RENAME” Strategy to Word Problems with Regrouping
- Addition With Regrouping From Ones to Tens and Tens to Hundreds
- Subtraction With Regrouping From Tens to Ones and Hundreds to Tens

Lesson 7: Introduce the “Put Together” Situation (Total Unknown): Apply the “FAST RENAME” Strategy to Word Problems With Regrouping
- Addition With Regrouping From Ones to Tens and Tens to Hundreds
- Subtraction With Regrouping From Tens to Ones and Hundreds to Tens

Lesson 8: Begin Generalization of the “Put Together” Situation and Continue Generalization of “Add To” and “Take From” Situations: Apply the “FAST RENAME” Strategy to Word Problems With Regrouping
- Addition With Zeros and Regrouping From Ones to Tens or Tens to Hundreds
- Subtraction With Regrouping From Tens to Ones and Hundreds to Tens

- Addition With Zeros and Regrouping From Ones to Tens or Tens to Hundreds
- Subtraction With Regrouping From Tens to Ones and Hundreds to Tens
- Addition With Zeros and Regrouping From Ones to Tens or Tens to Hundreds
- Subtraction With Regrouping From Tens to Ones and Hundreds to Tens

Lesson 10: Begin Generalization of “Compare” Situations and Continue Generalization of the “Put Together” Situation: Apply the “FAST RENAME” Strategy to Word Problems With Regrouping
- Addition With Zeros and Regrouping From Ones to Tens and Tens to Hundreds
- Subtraction With Zeros and Regrouping From Tens to Ones or Hundreds to Tens

Lesson 11: Continue Generalization of “Add To,” “Take From,” “Put Together,” and “Compare” Situations and Review Word Problems Containing Extraneous Information: Apply the “FAST RENAME” Strategy to Word Problems With Regrouping
- Addition With Zeros and Regrouping From Ones to Tens and Tens to Hundreds
- Subtraction With Zeros and Regrouping From Tens to Ones and Hundreds to Tens

Lesson 12: Complete Generalization of “Add To,” “Take From,” “Put Together,” and “Compare” Situations and Word Problems Containing Extraneous Information: Apply the “FAST RENAME” Strategy to Word Problems With Regrouping
- Addition With Zeros and Regrouping From Ones to Tens and Tens to Hundreds
- Subtraction With Zeros and Regrouping From Tens to Ones and Hundreds to Tens

Posttest
APPENDIX Q

Fidelity of Treatment Checklist
Fidelity of Treatment Checklist

Lesson: _____

Group: _____

For each instructional procedure included within the lesson, place a check mark in the corresponding box.

<table>
<thead>
<tr>
<th>Procedure</th>
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<tbody>
<tr>
<td>Advance Organizer</td>
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<tr>
<td>Describe and Model</td>
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</tr>
<tr>
<td>Guided Practice</td>
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<td>Independent Practice</td>
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<tr>
<td>Computation Practice</td>
<td></td>
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<tr>
<td>Score and Provide Feedback</td>
<td></td>
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</tbody>
</table>