5-2018

Effects of Dynamic Geometry Software on Secondary Students' Understanding of Geometry Concepts

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by

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A Starred Paper

Submitted to the Graduate Faculty of

St. Cloud State University

in Partial Fulfillment of the Requirements

for the Degree

Master of Science in

Curriculum and Instruction

May, 2018

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Chapter 1: Introduction

Research has shown that technology, ranging from calculators to advanced computer algebra systems and dynamic geometry software, supports students' ability to make and test conjectures about mathematical concepts (Forster, 2006). In a meta-analysis performed by Li and Ma (2010) it was found that "in general students learning mathematics with the use of CT (computer technology), compared to those without CT, had higher mathematics achievement" (p. 232). This technology has been shown to have a positive impact on both achievement and student affect towards mathematics (De Witte, Haelermans, & Rogge, 2015). However, other studies have found that computer technology, in any form, does not have an impact on student achievement or affect (De Witte et al., 2015).

The purpose of this study was to determine the effects of dynamic geometry software (DGS) on secondary students' understanding of geometry concepts. This paper examines how dynamic geometry software (DGS) is used in the secondary mathematics classroom and its effects on student understanding and achievement through a review of the literature. The use of technology in mathematics education is well documented and its effects are noted within this paper. The focus of the literature review in Chapter 2 includes: 1) how the computer technology is used in the classroom; 2) the use of DGS to promote student understanding of inductive and deductive reasoning; 3) creation of proof and the discovery of geometric theorems; and 4). the role of the software on student performance. In Chapter 3 I conclude by reviewing the research findings and giving recommendations.

Effects of Using Computer Technology in School Mathematics Classrooms

The use of computer technology in mathematics classrooms shows an increase in achievement among all grade levels K-12 when applied effectively (Li & Ma, 2010). Computer technology has a positive impact on student mathematical discourse, technical understanding, ability to correctly analyze data, and building a cooperative learning environment (Cooper, 2012; Forster, 2006). Li and Ma (2010) showed that an effective use of computer technology includes using a constructivist approach and that it has a significant effect on special needs students in particular. Forster (2006) showed that computer technology, specifically computer algebra systems, can help students gain understanding of syntax and interpreting data. Overall, computer technology, when used as an additional resource and under supervision of a teacher, can increase students' mathematical understanding across several subject areas and domains.

Constructivism, as defined by Li and Ma (2010), is an "approach of teaching as student-centered instruction that emphasizes strategies such as discovery-based (inquiry-oriented) learning, problem-based (application-oriented) learning, and situated cognition" (p. 219). Research by Li and Ma (2010) show that the use of computer technology with a constructivist approach has greater positive effects on student learning than a traditional approach. A traditional approach is defined as "teaching as teacher-centered whole-class instruction" (Li & Ma, 2010, p. 219). However, computer technology still shows positive effects on student learning and achievement even when used with a traditional approach (Li & Ma, 2010).

Computer technology also increases students' mathematical discourse (Cooper, 2012). Cooper (2012) maintained that technology can increase student engagement, class cohesiveness, cooperative learning, and writing in mathematics. Computer technology can extend students' thinking and provide real-world context which has shown to increase student motivation (Cooper, 2012). By increasing students' writing in mathematics through the use of computer technology, students must develop and reach higher levels of thinking in order to explain their reasoning to classmates and the teacher (Cooper, 2012). This increases student motivation and understanding while providing useful feedback to both the teacher and the student.

Dynamic Geometry Software

Dynamic geometry software is a computer program in which the user is able to construct and manipulate geometric figures. Dynamic geometry software (DGS) "allows users to construct geometric figures according to Euclidean principles and then dynamically alter them" (Hall & Chamblee, 2013, p. 14). Users can construct geometric figures using tools provided by the program and "drag" the figure, which will maintain its given properties, to make observations and predictions. This ability to dynamically manipulate figures saves time, provides a responsive visualization of an object's properties, and allows for immediate visual feedback to the user (Hall & Chamblee, 2013). These aspects of DGS enhance visual representation and spatial visualization, increase students' cognitive capacities during learning, encourage greater mathematical discourse, and pushes students' to become more mathematical thinkers (Crompton, Grant, & Shraim, 2018).

According to Weaver and Quinn (1999), visualization, modeling, discovering relationships, and problems solving are the four main geometric skills students should learn. Dynamic geometry software use in the classroom lends itself well to these skills. Students are able to use visual cues during manipulation to determine properties and propose theorems relating to those properties. Based on the construction, when points in the figure are moved, the measurements will adjust with the figure, which allows for observation of numerical patterns. The active engagement involved with constructing and manipulating a figure leads to greater student understanding when compared to students participating in a traditional lecture (Weaver & Quinn, 1999).

Despite the apparent panacea that DGS seems to offer, there are limitations. Teachers are reticent to use DGS due to the time it takes to create sketches, a limited background in knowledge and training in the use of the software, potential technological issues, and concern over the effectiveness of the use of DGS (Mitchell, Bailey, & Monroe, 2007). In a study done by Hannafin, Burruss, and Little (2001), two seventh-grade teachers and their students in ten 10 classes worked through two instructional programs; one a basics-first approach and the other an open DGS program. Hannafin et al. (2001), found that while most students enjoyed having control over their learning and were motivated to use DGS program, the teacher was concerned that the students had rushed through the problems and had not deeply understood the concepts.

In considering the potential effects of DGS on student understanding of geometry concepts, I questioned whether it was worth my time as a teacher to pursue a curriculum path involving these types of software. It is for this reason that I chose to explore this topic.

Research Questions

1. What effect, if any, does dynamic geometry software have on student understanding of geometry concepts?
2. Under what conditions, if any, is dynamic geometry software successful in enhancing student understanding of geometry concepts?

Focus of Paper

To answer these research questions, this study reviewed the literature on how dynamic geometry software is used in the classroom, the differences in the use of DGS versus not using DGS in the classroom on student learning, the use of DGS to promote student understanding, the role of the software on student performance, and external factors that affect the effectiveness of DGS. A literature review in Chapter 2 provides a basis for making recommendations and conclusions in Chapter 3. The literature review in Chapter 2 focuses on qualitative and quantitative studies on the use of DGS in and out of the classroom and its effects on students' thinking on geometry concepts.

Importance of the Topic

According to the National Council of Teachers of Mathematics (2000), students should be able to problem-solve, use reasoning to prove statements, communicate and connect concepts, and create and use representations to solve problems. These skills will transfer to "adult numeracy, financial literacy, and everyday life" (National Council of Teachers of Mathematics, 2000). Promoting and increasing these skills is necessary for economic stability, security, and encouraging a democratic society (National Council of Teachers of Mathematics, 2000). The main goal of geometry is to increase critical thinking skills. Dynamic geometry software has the potential to positively influence these skills.

Definitions of Terms

· Dynamic Geometry Software: computer programs which allow one to create and manipulate geometric constructions. The geometric constructions can be made to have certain properties, which can be tested and observed to make conjectures.

· Conjecture: a guess or idea based on observations of a pattern. In mathematics, a conjecture is like a hypothesis in that it is a reasonable guess based on observation, but it has yet to be proven true.

Chapter 2: Review of the Literature

The purpose of this literature review was to examine the use of dynamic geometry software within the classroom, the effects of the software on learning geometry concepts, and the software's effects on student performance. In Chapter 1, the background information on computer technology and dynamic geometry software was introduced. This chapter is organized into five main parts: DGS use in the classroom, the role of DGS on inductive and deductive reasoning, effects of the software on student performance, and the limitations and differing viewpoints of DGS.

Use of Dynamic Geometry Software in the Classroom

Guven, Cekmez, and Karatas (2010) described DGS as "dynamically manipulable interactive geometry software" (p. 193). One of the main features of DGS is its ability to drag figures or parts of figures and maintain the properties the figure was originally constructed with (Guven et al., 2010). The use of DGS within the classroom ranges from a teacher-centered, demonstrative, traditional approach to a student-centered, investigative, constructivist approach.

Ruthven, Hennessy, and Deaney (2008) explored four case studies involving both traditional and constructivist approaches to DGS. Ruthven et al. (2008) found that the software was flexible enough to fit both styles of teaching. Teachers with time constraints were able to pre-construct figures that students could manipulate to observe patterns and make hypotheses about their observations. Teachers could also lead a whole class discussion while displaying a figure and manipulating it themselves (Ruthven et al., 2008). In a more constructivist approach, teachers created guided discovery activities that helped lead students through constructions (Ruthven et al., 2008). Difficulties with the software were dealt with differently under each approach. In the traditional approach, teachers sought to hide any imperfections or potential misleads caused by DGS (Ruthven et al., 2008). This was for the sake of time and to prevent students' confusion. In the constructivist approach, teachers used difficulties with the software as learning opportunities (Ruthven et al., 2008). They challenged students to critically think about why the mistake was happening and how to fix it. This led to an increase in understanding of mathematical syntax by students (Ruthven et al., 2008). Teachers that used the constructivist approach gained deeper insight to their students' understanding of different mathematical properties.

Connor, Moss, and Grover (2007) explored the use of DGS in investigative activities with pre-service teachers. While this study focused on post-secondary use of DGS, the findings can reasonably be applied to secondary teaching. In this case, students were given an "if...then" statement and asked to explore the meaning of the statement and construct a formal proof (Connor et al., 2007). Students were inexperienced with the software and made incorrect justifications in their proof or reversed the hypothesis and conclusion of the statement (Connor et al., 2007). Connor et al. suggested that the software is limited when students are not guided by an instructor to help lead them through common mistakes and misunderstandings. Teacher presence and guidance influences the effectiveness of the software.

Hollebrands (2007) explored the reactive and proactive strategies students use when provided with DGS technology. DGS promoted reactive strategies by students who may not have known how to proceed with a proof or construction and simply tried different tools to progress. Students required prompting from the teacher in order to identify mathematical relationships. In comparison, with teacher guidance, students were more likely to use proactive strategies that were characterized by critical thinking and purposeful manipulation within the DGS technology. This corroborates with Connors et al.'s (2007) findings that DGS technology is limited without teacher guidance.

Dove and Hollenbrands (2014) explored the use of DGS technology among three secondary geometry teachers. The software was utilized by all three teachers to provide scaffolding in understanding specific mathematical concepts.

Students were given guided activities to construct and manipulate figures. The teachers differed on student collaboration in that some activities were done in pairs or small groups and some were done individually. In either case students were observed discussing their findings and conclusions with each other. The teachers also worked with students during the activities and provided feedback to individuals and groups. This time also provided feedback to the teachers on student misconceptions and understanding. It was noted that "all three teachers felt that GSP (Geometer's Sketchpad) was allowing them to improve the cognitive demand and conceptual thinking of their class" (Dove & Hollenbrands, 2014, p. 679). The use of DGS technology promoted mathematical discourse among students and provided meaningful feedback to both the teachers and the students.

The use DGS within the classroom is still at its beginning stages. While the software is flexible enough to be employed with traditional instruction and constructivist instruction, Li and Ma (2010) showed that a constructivist approach is more likely to produce noticeable gains in student learning and achievement. Constructivist uses of DGS include guided discovery, inquiry-based learning, and bridging the gap between inductive reasoning and formal proof. Given some of its inherent difficulties, DGS is still valued for providing figures that are accurate, relatively quick to make when compared to the paper and pencil method, and easily manipulated to provide immediate feedback (Ruthven et al., 2008).

Role of the Software in Bridging Empirical and Deductive Reasoning in Mathematics

Empirical justification is “characterized by the use of examples as the main (maybe the only) element of conviction” in determining the veracity of a conjecture (Marrades & Gutierrez, 2000, p. 91). Deductive justification is “characterized by the decontextualization of the arguments used, are based on generic aspects of the problem, mental operations, and logical deductions, all of which aim to validate the conjecture in a general way” (Marrades & Gutierrez, 2000, p. 93). Students have difficulty in bridging the gap between these two types of justification. Marrades and Gutierrez (2000) showed that DGS helps students by providing a manipulable figure that can be easily tested for patterns and provide multiple representations very quickly. DGS shows students why formal proofs are important by displaying common misconceptions associated with empirical justification. It also eases the transition between empirical justifications to deductive justifications in a way that is difficult to do with traditional approaches to teaching proofs.

Jones (2000) provided further evidence that DGS can extend student thinking beyond empirical or inductive reasoning to deductive reasoning. Findings show a gradual change in student thinking from descriptive language in the beginning stages of using DGS to mathematically precise language over a 9-month period (Jones, 2000). The linking stage between these two types of thinking were “influenced (mediated) by the nature of the dynamic geometry software (for example by the use of the term 'dragging' or by other phrases linked to the dynamic nature of the software)” (Jones, 2000, p. 80). The drag feature is a key influence on bridging students' thinking from an inductive nature to deductive nature. It provides easy to see empirical evidence that over time can help guide students to understanding properties and applying these observations to form mathematical reasoning for explaining the features of a figure (Jones, 2000).

In a study done by Leung, Baccaglini-Frank, and Mariotti (2013) it was found that DGS provides unique feedback that can guide learners to revealing different geometric properties. Students' perception of different patterns of variation such as contrast, separation, generalization, and fusion can lead to intentional discovery of geometric properties (Leung et al., 2013). The dragging feature afforded by DGS can reveal contrasts in different properties of a figure as well as separate which properties remain under certain conditions and which change (Leung et al., 2013). Generalizing these observations leads to a fusion of concepts which results in a conditional statement. This thought process is the premise of learning proofs and critically thinking through observations and patterns to create a conjecture, a main theme of geometry.

In an exploratory case study done by Arzarello, Bairral, and Danè, (2014) on five high school students in Italy using DGS with a touchscreen the researchers were able to identify which types of manipulation were performed on the figure and in what order. Basic actions, such as tapping and holding to select objects, and active actions, such as dragging flicking, freeing, or rotating were observed. Two domains of manipulation appeared: the constructive domain and the relational domain. After constructing a figure, students spend their time dragging, zooming, and rotating to determine relationships between the parts of the figure. The authors believed that through direct contact on a touchscreen, DGS adds another layer of feedback that promotes discovery and engagement (Arzarello et al., 2014).

A case study on a ninth grade geometry student by Lachmy and Koichu (2014) found relationships between empirical and deductive reasoning in proving “if and only if” statements. “If and only if” statements are biconditional statements “used to introduce a condition which is necessary as well as sufficient” (Oxford Dictionaries | English, 2018). Using DGS to explore a geometric theorem, it was found that the student used deductive arguments to support the justification of an “if” statement and used empirical arguments to support the “only-if” statements. The student ascended from empirical to deductive proof when proving the “if” statement. However, when exploring the “only-if” statement, the student used only empirical evidence to convince herself that the statement was true.

Here we can see how DGS can support the transition from empirical-based to deductive-based reasoning under certain conditions. We can also see how the rapid visualization of DGS can hinder growth from empirical to deductive proof. The authors noted that this hindrance may be caused by students’ common mistake of confusing a statement and its converse (Lachmy & Koichu, 2014). Only when the DGS was designed to linearly support the original statement did the student move from empirical to deductive reasoning. This supports the claim that DGS can most effectively be utilized under strict guidance and intentional support.

Role of the Software on Student Performance

In a study performed by Patsiomitou (2008), it was found that students in an experimental group with access to DGS outperformed students in a control group without access to DGS. In combination with real-world problems and a DGS enriched environment, students in the experimental group were able to formulate higher-level reasoning to solve a given problem and provide a proof where their control-group peers were unable to get so far as to solve the same problem (Patsiomitou, 2008). Students were comparable in age, gender, and van Hiele level; however, the sample size was small with a total of 28 students, fourteen in each group. It was found that other factors affected students' ability to reach higher level thinking including pre-existing knowledge of theorems (Patsiomitou, 2008). While this was a small, qualitative study, it has strong implications for the effectiveness in increasing students' problem-solving and deductive reasoning skills.

In a study performed by Funkhouser (2002), students were compared over a 36-week period with an experimental group being taught under a constructivist, computer-based (DGS) approach and a control group taught under a nonconstructivist, noncomputer-based approach. The 27 students in the control group and 22 students in the treatment group were compared with a t-test to observe differences in geometry performance scores and student attitude toward mathematics (Funkhouser, 2002). It was found that the experimental group outperformed the control group in geometry performance at the 0.05 level of significance, with a higher mean score and a smaller standard deviation (Funkhouser, 2002). The attitude assessment showed a difference only for the control group between the pre- and post-test, namely within the control group which increased its agreement with the phrase "One of my best subjects is mathematics." (Funkhouser, 2002, p. 170). It was found that students taught using a constructivist, computer-based approach increased their performance in geometry, but did not foster a more positive attitude for mathematics (Funkhouser, 2002). This provides further evidence for the case that DGS technology can increase student performance.

Pitta-Pantazi and Christou (2009) explored the relationships between cognitive styles, DGS, and measurement performance in a study done on 49 sixth-grade students involving areas of triangles and parallelograms. An intervention program which featured DGS was shown to improve the performance of all students, regardless of cognitive style. The authors noted, however, that there were multiple factors that positively influenced student performance including teacher instruction and student interaction. It was noted, however, that while all students improved in their performance, students that prefer verbal processes over visual processes significantly improved their performance in the construction items on the test compared to the other groups. This was unexpected by the authors as it was thought that DGS would better fit with the visual thinker group's cognitive style. The results of this study offer insights into the potential impact of DGS on different thinking styles (Pitta-Pantazi & Christou, 2009).

Isiksal and Askar (2005) compared the performance of 64 seventh-grade students under differing conditions including a control group, a spreadsheet-based instruction group, and a DGS-based instruction group. It was found that the DGS group and the traditional (control) group had significantly greater mean scores in mathematics achievement compared to the spreadsheet group. The study compared self-efficacy scores and performance as well and found that students positive affect towards the DGS instruction could have led to higher scores (Isiksal & Askar, 2005). Further research is required to determine what had the greatest effect.

Limitations and Differing Viewpoints of Dynamic Geometry Software

Much of the research on DGS supports the benefits of the software on students' learning. There are opposing viewpoints that claim that DGS can inhibit students' learning. Marrades and Gutierrez (2000) explained that DGS may decrease students' use of deductive justification because DGS promotes empirical justification. The design of the software does promote empirical exploration and without meaningful teacher intervention students may not increase their deductive reasoning skills (Marrades & Gutierrez, 2000).

There are many limitations to the software for classroom use. Teachers cannot allow students to construct every figure and build their deductive reasoning fully due to time constraints (Dove & Hollenbrands, 2014). Students must also learn to be fluent with the software before they are able to use DGS to perform higher level thinking tasks, which takes time. Teachers are also somewhat limited in their training and comfort level with the software (Ruthven et al., 2008). This limits its use in classrooms as a whole and limits its use by students, even though research shows a student-centered approach to be more effective (Ruthven et al., 2008). Among pre-service teachers, findings show a limited understanding of the software and the ability to apply higher level thinking tasks (Guven et al., 2010). Without teacher fluency in the software, it is unlikely the potential for the software will be reached.

In a series of qualitative case studies by Norton, McRobbie, and Cooper (2000), five secondary teachers from a technology-rich private school were interviewed and surveyed to determine the teachers' attitudes and practice of using computers in teaching mathematics. It was found that the use of computers in mathematics teaching was a low priority, should be used to support traditional, lecture style teaching, and did not support content-focused pedagogy. Teachers were wary of using computers as they were concerned about losing control of the classroom and not covering the necessary material. They were more assessment oriented, which prevented a change in teaching into a more constructivist approach. In a situation where access to technology was not an issue, teachers were still unwilling to try to incorporate computer technology into their teaching.

While this study was done over a decade ago, more current studies support the observations reported by Norton et al. (2000). Howard, Chan, and Caputi (2015) looked at subject areas and teachers' integration of laptops in secondary teaching. When comparing the use of laptops in English, mathematics, and science it was found that mathematics reported the lowest frequency of integration, least confidence in teacher readiness to use laptops, and significantly less positive beliefs about the importance of information computer technology over the course of 3 years. This indicates a seemingly cultural reticence toward using technology in mathematics. Without a change in practice, any potential for DGS to improve student understanding of geometry on a large scale is unlikely.

Summary

The findings of these studies indicate generally positive outcomes of DGS promoting student understanding of geometry concepts. However, DGS is dependent on well-thought intentional use by a teacher to promote student success. The thought process students use when working with DGS has been documented and shows promise towards increasing critical thinking and problem-solving skills. Limitations of the software may prevent consistent use by teachers. Chapter 3 explores my conclusions and recommendations.

Chapter 3: Conclusions and Recommendations

The purpose of this research paper was to determine what effects dynamic geometry software has on student understanding of geometry concepts. Chapter 1 included background information on technology use in the classroom and what dynamic geometry software looks like. Chapter 2 reviewed the literature on this topic. In Chapter 3 I conclude with a summary of my findings and make recommendations.

Conclusions

I reviewed articles on the use of dynamic geometry software in the classroom, its role in bridging types of reasoning, its effects on student performance, and the limitations of the software. Most of the research supports the use of DGS in promoting student understanding of geometry concepts. However, the software is most effective under certain conditions including a constructivist approach and the use of well-thought out lessons designed to guide students along a path of discovery. Without proper training and encouragement, teachers may not use the software at all due to time constraints and difficulties in adapting to a new teaching style.

Much of the research focuses on the effects of DGS beyond student performance on written assessments. Many of the studies employed DGS as a way of analyzing students' thought process on discovering a theorem or making a conjecture. DGS provides a tangible way for researchers and teachers to explore students' thinking. Whether on a computer or a touch-screen the DGS provides another way for students to represent their thinking beyond what was previously possible with paper and pencil. This ability to perceive the thought process of students is unique to DGS in that not only can researchers observe the static imagery students produce, they can also observe the actions students take over time and in what order to determine the reasoning behind their conjectures.

Other effects of DGS include producing a more positive affect toward geometry and an increase in mathematical discourse. Research has shown a "positive correlation between self-efficacy and mathematics" (Isiksal & Askar, 2005). It is undetermined if the positive reception of DGS by students is due to the inherent nature of the program or because it is a novelty. Studies report that students enjoy the autonomy DGS can give them as well as the interactive nature afforded by the software. Questions arise as to whether it is the constructivist approach that is causing the increase in performance or the software. Would a constructivist classroom that did not use DGS perform as well or better than a constructivist classroom with DGS? Further research is required.

Many of the studies had students work in small groups or pairs, with just a few working individually. The DGS provided an opportunity for students to develop greater mathematical syntax and mathematical language. Explaining one's thoughts or ideas to another was helped by the software's representations of one's ideas. This was observed throughout many studies. Even in whole-class instruction, the teacher's ideas appeared to be better understood by the students with the visual aid of DGS.

Performance reviews of DGS show promise for its application in increasing student understanding of geometry concepts. Many studies showed in increase in achievement when DGS was coupled with other strategies. This seems to indicate that DGS could enhance teaching and learning, but alone cannot improve performance.

Recommendations

While the results of DGS were generally positive, there are limitations to the software. More research needs to be done on the effects of DGS alone without the aid of other strategies. Concern over the practicality of DGS also arose in several studies. If teachers are unwilling to use DGS due to time constraints or unfamiliarity with the software its effects will not be purposeful. How and why teachers do and do not use DGS could be explored further.

The effects of DGS is amplified under a constructivist approach with intentionally designed lessons that guide students' thinking. In order to increase the use of DGS, training and creation of such lessons is required. However, even in a traditional-style classroom DGS can enhance student learning. The ability to visualize a theorem in a dynamic way can have positive effects under most conditions. So, even if a teacher is unwilling to change from traditional to constructivist style teacher, the use of DGS is still a valid strategy.

Further research on the long-term effects of DGS is required. Many of the studies took place over a few weeks. While the short-term effects seemed positive, whether DGS had a long-lasting effect on student understanding is unknown. Part of the allure of DGS is the idea that it cannot only provide a different means of learning, it can cause deeper understanding of a concept which will result in better long-term learning. The studies that I read did not explore this and so it is unknown if DGS provides more in-depth knowledge.

Within my own classroom, I believe there is enough evidence to support the use of DGS. The results of this study will be used to inform and advise the math department within my own school. As part of the curriculum team within my school, I will also use the results of this review to explore materials that correspond with the use of DGS in the classroom.

Summary

DGS has the potential to increase student performance and affect toward geometry. Under specific, but reasonable, conditions DGS can result in greater student understanding of geometry concepts. However, the limitations of the software and supporting material are the main reasons for its non-use. It is also undetermined what the long-term effects of DGS are. Future research should focus on what effect DGS alone has on student understanding in the short- and long-term and if the limitations of the software can be overcome.

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