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Effective Interventions to Increase Basic Math Skills for Students with Learning Disabilities

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Effective Interventions to Increase Basic Math Skills for Students with Learning Disabilities

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Chapter 1: Introduction

According to the National Center for Learning Disabilities (NCLD, 2017), students with learning disabilities are three times more likely to drop out of school–18.1% compared to 6.5% for the average dropout rate of all students. Dropping out of school is a severe problem for the students, communities, and the nation. Although recently the dropout rate is not high compared with the past decade, there are many students who cannot graduate secondary school. According to data from the National Center for Education Statistics (NCES), the overall tendency of dropout rates of 16- to 24-year-olds who are not enrolled in school and have not earned a high school credential such as a diploma or GED certificate decreased from 2006 to 2017. However, there was 2.1 million student dropouts, and the overall student dropout rate was 5.4% in 2017, which was not a small amount. Moreover, the rate of dropping out of school of students with learning disabilities is even higher than students without disabilities. Many studies have reported that American students have lower math achievement than other countries (DeSilver, 2017). In general, mathematics is the most challenging subject and the majority of students with and without disabilities hate mathematics all over the world (Gafoor & Kurukkan, 2015). According to Scarpello (2007) reports, 75% of Americans stopped studying math and chose job that stay away from the math even though we are using math everywhere in our lives (Gafoor & Kurukkan, 2015). The ability to think mathematically is a crucial skill in an increasingly competitive job market; the demand for mathematics-intensive science and engineering jobs grows outstandingly in overall job growth three to one (National Mathematics Advisory Panel, 2008).

Math is the most difficult subject for students with learning disabilities who have difficulties in various subjects. Most American students think that math is hard, boring, and complicated because there are many formulas (NCES, 2007). To solve mathematical problems, they must apply the formula and calculate it. In this process, many students with learning disabilities experience difficulties in calculating natural numbers by hand. Basic mathematics skills with natural numbers such as addition, subtraction, multiplication, and division are fundamental skills, and are used in the real world. Math is not only a science of numbers, but is also used in everyday life from calculating time and distance to handling money and analyzing data to make decisions in financial planning and insurance purchasing. It is also essential in the STEM (science, technology, engineering, and math) fields (Soares et al., 2017). Therefore, students should learn and have proficient skills with calculating natural numbers in order to function in everyday life (Faith, 2018).

Research Question

The following research question guides this review of literature:

- What are effective interventions to increase fundamental math skills for students with learning disabilities?

Historical Background

To start, the terminology to refer to someone with difficulty learning has contextually changed over time. Learning difficulties have not always been referred to by the term 'learning disabilities'. In 1877, the term 'word blindness' was coined by German neurologist Adolf Kussamaul to describe "complete text blindness, although the power of sight, the intellect, and the power of speech are intact". This was the first term used to relate what we now refer to as learning disabilities. Other terms that have been used over the years have been dyslexia, developmental disorders, cognitive disabilities, intellectual disabilities, and learning deficit. Kirk (TreeHozz, 2020) was the first person to use the term learning disabilities at a conference in Chicago. After 6 years, Congress passed the Children with Specific Learning Disabilities Act, which was included in the Education of the Handicapped Act of 1970 (PL 91-230). This was the first-time federal law mandated support services for students with learning disabilities. The Education for All Handicapped Children Act (PL 94-142) mandated free, appropriate public education for all students and was passed in 1975. This law was renamed the Individuals with Disabilities Education Act (IDEA) in 1990. As the law was renamed as IDEA, the term "handicapped" used in Public Law 94-142 had been changed to the term "child with a disability" in its statutes and regulations. Also, the new law required transition services for students and at that time, autism and traumatic brain injury were added to the eligibility list. Later in 2004, IDEA was reauthorized again in alignment with the No Child Left Behind Act, giving school personnel more authority in special education placement decisions.

There are many types of learning disabilities; mathematical disability being one of these types. Mathematical disability (MD) is a relatively promising field compared to other types of learning disabilities. MD is known as dyscalculia, which refers to difficulty in learning or comprehending arithmetic, such as difficulty in understanding numbers, performing mathematical calculations, and learning facts in mathematics. Approximately 7% of school-aged children have a learning disability (LD) in mathematics (Geary et al., 2012). This percentage is problematic because math skills are necessary to build a foundation for everyday life skills. They are also taught and tested at all grade levels, which establish it as a core subject. Recently, math is not just considered a subject itself, but can also be applied to many other subjects, such as science, technology, and engineering. Math is the common denominator among these subjects. Having sufficient skills in math is increasingly important because math, both in real-life and in relation with other subjects, is becoming more necessary.

Focus of Paper

The number of students aged 3 to 21 who received special education services under the Individuals with Disabilities Education Act (IDEA) in 2019–2020 was 7.3 million, accounting for 14% of all public-school students. Among students receiving special education services, specific learning disabilities (33%) were the most common category of disabilities (NCES, 2020a, b).

Researching effective interventions is important for students with learning disabilities (LD) to help them develop their basic math skills. I am researching three effective interventions for students with learning disabilities from elementary to secondary school. In the mathematics curriculum of elementary school, children learn how to add, subtract, multiply, and divide with natural numbers. In secondary math curriculum, students learn to calculate with integers, rational and irrational numbers, and imaginary numbers. Among these different types of numbers, integers have a wider range of numbers than natural numbers, a more complicated operating system compared to the natural one because it contains negative numbers. Thus, before children learn more complicated skills, they must have a flawless technique with addition, subtraction, multiplication, and division with natural numbers and these mathematical skills are often used in real life. There are real life uses of math, such as children finding the total price of items that they put into the shopping cart in the grocery store, or they can save money through checking how much money is left in their bank account using basic addition and subtraction. Since these skills are important in making a living, students need to have a good grasp of natural numbers. Therefore, researching effective methods is so important to teach fundamental skills to students with learning disabilities.

Importance of the Topic

Researchers have stated that if children do not learn mathematics skills at an early age, they might continue to have difficulties in mathematics throughout their whole school years (Arnold et al., 2002). The converse is probably also true; if young children learn and grow their abilities to solve mathematics problems, they are more likely to succeed in math throughout their secondary school years. If children are confident in math and proficient at solving math problems, they can take more advanced courses in mathematics and science. This may be true even if a student has a learning disability. Being successful in math can also encourage a higher level of motivation; having a high level of motivation is important for children. Adelman and Taylor (1983) suggested that typically students with disabilities have low motivation to learn. Specifically, students with learning disabilities have low confidence in math and they do not know the reason they need to study this subject in school. Students can obtain a higher level of motivation through practicing and learning math in effective ways. Saffer (1999) mentioned being good at math is directly linked to successful employment after high school graduation. In addition, for those who master basic algebraic math skills, they may be able to learn more advanced mathematical topics (Witzel et al., 2008).

Definition of Terms

Throughout this paper, I will be using several terms to describe key vocabulary such as learning disability, math disability, intervention, evidence-based practice, and cognitive strategies. I have provided definitions below as they relate to literature in the field. These definitions will be used for the terms in this review of the literature.

Specific Learning disability (LD), also referred to as a learning disorder, is defined by the American Psychiatric Association (2013) in the Diagnostic and Statistical Manual of Mental Disorders (5th ed.) as a disorder which:

> comprises a heterogeneous group of disorders characterized by persistent difficulties with learning academic skills in a variety of domains, including reading, spelling, written expressions, and mathematics. The symptoms of specific LD must have persisted for at least 6 months, even though interventions that target those difficulties were provided. Furthermore, the affected academic skills must be substantially and quantifiably below levels expected for the person's age and cause interference with academic or occupational performance or with activities of daily living (based on a clinical synthesis of the individual's history, school reports, and psychoeducational assessment). The learning difficulties are not accounted for by intellectual disabilities, by uncorrected problems with visual or auditory acuity, or by lack of language proficiency, inadequate educational instruction, or psychosocial adversity. The academic domains and subskills that are impaired are specified within each of the following domains: reading (word reading accuracy, reading rate or fluency, reading comprehension), written expressions (spelling accuracy, grammar and punctuation accuracy, clarity, or organization of written expressions), and mathematics (number sense, memorization of arithmetic facts, calculation fluency or accuracy, accurate math reasoning). pp. 66-74)

Mathematics disability (MD) is a neurologically based difference in how an individual processes numerical information, which leads to significant difficulties in learning and doing mathematics (Butterworth, 2010).

Intervention, also referred to as a response to intervention (RTI), is a procedure used by teachers or educators to help students who are struggling with a skill or lesson. It addresses the conceptual and procedural bases for emerging competence with arithmetic (Fuchs et al., 2019).

Evidence-based practices (EBP) are defined as “practices that are supported by multiple, high-quality studies that utilize research designs from which causality can be inferred and that demonstrate meaningful effects on student outcomes” (Cook & Cook, 2013, p. 73).

Cognitive strategies are one type of learning strategy to help students to organize and process information. These include repetition, organizing a new language, summarizing meaning, guessing the meaning from context, and using imagery for memorization.

Chapter II: Review of the Literature

There have been many studies on effective teaching strategies to learn basic math skills for students with learning disabilities. This chapter is organized into three major parts: (1) studies that using technology models, (2) studies that review the effectiveness of number line models, and (3) studies that examine the effectiveness of Concrete-Representational-Abstract (CRA) models. In total, ten studies were chosen for review that evaluated the effectiveness of interventions for improving mathematics performance in students with learning disabilities. Table 4 presents these studies in the same chronological order in which they appear in Chapter II.

Technology Models

Bowes (2010) examined technology's place in mathematics curricula. In this study, Bowes stated, "Technology supports achievement, enabling learners to be independent, competent, and creative thinkers, as well as effective communicators and problem solvers" (p. 1). He stated that technology can assist students with relating concepts to real world experiences, help in accurately computing solutions, and enhance math state standards (Bowes, 2010). I researched three educational technologies; Computer-based Math Fluency (CBMF), usage of calculator, and iPad that assists to increase basic math skills for students with LD.

Computer-Based Math Fluency (CBMF)

Many schools use computer-based interventions to ensure efficient delivery (Jimenez et al., 2003), and this kind of intervention involves interaction between students and computers with little supervision by teachers or interventionists. Previous research related to computer-based interventions found that it increased not only math skills (Holmes et al., 2006; Springer et al., 2007; Ysseldyke et al., 2005) but also the efficiency of the delivery of a tiered intervention model (Ysseldyke & McLeod, 2007). Moreover, it made easy to access, manage, and analyze the data using technology. However, previous studies focused on general math proficiency rather than on a specific National Research Council area of math proficiency and they did not focus on students with learning disabilities (Burns et al., 2012). Ysseldyke and colleagues (2005) studied a computer-based math intervention by examining Math Facts in a Flash (MFF) (Renaissance Learning [RL], 2003). MFF was one of the computer-based math interventions and it was designed to increase computational fluency with independent practice with math facts. The study found that students who participated in MFF significantly overperformed and more than 90% of the teachers interviewed responded that they liked the intervention. However, the results were from students in the general population, and they were not identified as at risk for math difficulties (Ysseldyke et al., 2005).

Burns et al. (2012) examined the effects of a computer-based math fluency (CBMF) intervention on the math abilities of students with mathematics disabilities (MD) in third and fourth grade. One hundred forth-five third graders and 86 fourth graders participated in this intervention. Researchers used Math Facts in a Flash (MFF) as an intervention tool and conducted interventions at least three times per week for 8 to 15 weeks. MFF is a software program designed to enhance computational abilities in four basic mathematics operations: addition, subtraction, multiplication, and division (Burns et al., 2012) and fluent computation was significant math goal (NCTM, 2000a, b; NMAP, 2008) for students identified as at risk for math difficulties (Geary et al., 2007; Hanich et al., 2001).

The data were obtained from 231 students in third and fourth grades. Students spent approximately 5 to 15 minutes working independently on solving math problems during each intervention session. Each session was composed of 40 math problems, which appeared on a computer screen in a large font, such as one problem per screen followed by three choices of answers under the thick line below each problem. Students could see a red circle on the problem if they selected the correct answer and were allowed move on to the next problem. On the other hand, a red circle crossed-out with a large X mark would appear automatically if they chose the incorrect answer. When they completed their intervention session, the results were displayed on the screen to show how long it took and how many problems the students correctly completed. The problems of MFF consisted of one of 62 hierarchical levels consisting of between 11 to 15 levels per grade. Each practice set consisted of a randomly determined set of problems from the level that the student was working at, and the student would be given up to 2 minutes to finish the practice set. If students completed 40 problems with all correct answers within a limited time, then the students' level was mastered, and they could start the next level of the session. During the process of this math intervention, the students essentially completed a baseline test to decide the appropriate set of problems (e.g., multiplication by 4 and 5) and then practiced the multiplication fact from that set of problems until they could correctly complete all problems within 2 minutes. When the students mastered a specific set of problems, they moved on to the next skill level in the hierarchy (e.g., multiplication by 6 and 7). Star Math (RL, 2002), a computer-adapted assessment system, was used to collect data for screening and pre- and post-intervention (Burns et al., 2012).

The intervention sessions were supervised by classroom teacher even if MFF was mostly self-sufficient with little required supervision. The classroom teacher assisted starting the program and organized computers, and they also provided additional verbal cues to keep students on task. However, they did not give any feedback or instruction on the math problems during the intervention (Burns et al., 2012).

As a result of this study, the invention of CBMF was a successful tool to increase the math skills of students with MD. The research found that increased repetition led to enhanced recall and fluency of the skill (Burns, 2005; Daly et al., 2000; Szadokierski & Burns, 2008). The data were analyzed by ANCOVA that used the growth NCE as the dependent variable and the pretest NCE as the covariate. Moreover, Cohen's (1988) *d* was computed to estimate of effect size and a χ^2 nonparametric test was used to compare the percentage of students identified as at risk after the intervention between the two groups. To determine significance, a corrected alpha level of .025 was used (cited in Burns et al., 2012).

The results showed that the students who participated in the CBMF intervention had higher scores in Star Math than those in the control group. The pretest NCE score for the two groups was not significantly different for third grade, $t(279) = 1.00, p = .32$, or fourth grade, $t(159) = 1.62, p = .11$, which resulted in small effects of $.12$ and $.26$, respectively. Moreover, after CBMF intervention, 42.8% of the third graders who participated in the intervention scored above the 25^{th} NCE, while 30.6% of the students in control group did so, although all participants scored below the 25^{th} percentile on the assessment before conducting intervention. Among the fourth graders, 42.5% of the participants who received intervention scored at or above the 25^{th} NCE, but 29.1% of the students in control group recorded at or above 25^{th} NCE.

The result showed a significant effect as $\chi^2(n = 60, df = 1) = 9.86, p < .025$. Therefore, current findings suggested that CBMF intervention could be an effective factor to improve basic math skills of students at risk of math difficulties and students with LD (Burns et al., 2012).

Calculator

Calculators are a common tool in math lessons and even in real life. Calculators are also the most widely used accommodation for students with disabilities. Although using a calculator for students with disabilities is a commonly used accommodation in general, there is little research that exists on students with disabilities and calculator use (Maccini & Gagnon, 2005). As Maccini and Gagnon reported, however, calculators are a useful mathematical tool to show the result of calculated natural numbers and integers instantly. There is a study about the effects of calculator usage in learning mathematics for students both with and without disabilities.

Bouck et al. (2013) examined students with and without disabilities using calculators to solve the problems of mathematics assessments and whether using calculators helped their performance. One hundred forty-six 6th graders and 149 7th graders participated in this study. Students who attended inclusive mathematics classes and a diverse group of students with disabilities (e.g., students with LD, hard of hearing, ADHD) were included in the list of participants in both grades. 11 teachers (seven in sixth grade and four in seventh grade) were involved with the study as well (Bouck et al., 2013).

The study measured data collection from assessment, which emphasized two of the National Council of Teachers of Mathematics (NCTM, 2000a) content area strands–number operations and algebra–for both sixth and seventh grade participants. The assessment consisted of multiple choice and open-ended problems to check not only their calculator usage, but also evaluated students’ understanding of concepts (Post et al., 2008). All students completed 16 assessments spread across 9 months in one school year. They were given 10 minutes to finish each assessment. They also marked “c” next to the problems, when they used a calculator to solve the problems (self-reported data) (Bouck et al., 2013).

Data were analyzed by SPSS databases to reflect calculator use and correctness on each question across assessments. The researchers analyzed the data of students’ self-reported calculator use. Chi-square was also used to see if there was a relationship between using a calculator and answering problems correctly (Bouck et al, 2013).

The data from Table 1 indicated that students with and without disabilities answered a greater number of problems correctly when they used a calculator. Besides, the students with disabilities in sixth grade got more correct answers with a calculator than students in seventh grade. Consequently, all students were more successful performance with a calculator (Bouck et al., 2013).

Table 1

Chi-square Test of Association Data-comparing Calculator to No Calculator

Groups	All students	Students with disabilities
Sixth grade		
Percent correct with calculator	75.5%	79.4%
Percent correct without calculator	49.6%	17.3%
Seventh grade		
Percent correct with calculator	73.8%	59.5%
Percent correct without calculator	49.4%	49.9%

In summary, the students with and without disabilities got more correct answers when they used calculators. The students used calculators not only for finding answers, but also for checking answers, trying strategies, or reducing the cognitive parts in mental mathematics problems. This study suggested students both with and without disabilities could get more correct answers with a calculator and it gave support for greater consideration of calculator use. It also demonstrated how a calculator can act as a cognitive prosthesis for students with disabilities (Edyburn, 2005; Rapp, 2005). Besides, using calculators in that manner will be a greater consideration for them (Bouck et al., 2013).

iPad

Kaur et al. (2017) examined the effects of using an iPad as a supplementary teaching tool for students with learning disabilities. Zhang et al. (2015) reported from an exploratory study about the effects of iPad apps on math skills of students and they found the apps improved student learning and decreased achievement gaps between the students were struggling and the students who were not. iPads are also a very effective tool to improve literacy skills (Beschorner & Hutchison, 2013), therefore, it can enhance student motivation, promote independence, provide opportunities for self-expression, (Flewitt et al., 2014) and improve engagement (Hutchison et al., 2012).

This study planned a 5-week service-learning project and one-on-one math tutoring services using an iPad provided to participants. Ten students participated in this study, including three African American females, four African American males, and three Caucasian males. Ten teachers were selected from different grade levels from early childhood to middle school. They had successfully completed a technology course and were trained on the way to use different types of technology in K-12 classrooms before starting services (Kaur et al., 2017).

The teacher candidates had to prepare lesson plans including the math topics to be addressed each week and the apps that would be used with the particular reasons. They also had to reflect on their tutoring related to the effectiveness of apps and experience. During the tutoring services, the teachers downloaded and used free apps such as Chalkboard, Division for Kids, Division Wiz, Grade 4 Math, iTouch, Math Animations, Number Frames, OoO Calc, Splash Math, and Y Homework to meet the math standards and topics. At the end of the project, teachers were required to fill out the open-ended survey regarding their experience of using math apps on iPad with their students (Kaur et al., 2017).

During the tutoring sessions, each teacher taught one student and using apps in iPad with the traditional teaching methods for 5 weeks. Teachers used iPad apps when they were teaching and practicing with students to solve math problems. The problems had step-by-step instructions with explanation and answers including different ways to solve. When students were not able to solve them, the teacher gave them an extra explanation available in app through step-by-step method to help understanding. This process let the students know the steps they were missing or were confused about and helped them to understand the problems better. Through the usage of iPad, it demonstrated the iPad not only helped with the activities that the students could do, but also showed step-by-step instructions on many apps (Kaur et al., 2017).

As a result of the project, adding iPads to traditional teaching methods improved basic math skills, such as conceptual understanding of numbers, order of operations, expressions, and multiplication and division skills for students with learning disabilities. Content-specific apps kept students engaged, focused, and motivated because the apps were used in alignment with the learning abilities of each student. For example, iPad apps “Division Wiz” and “Math Animations” broke down the steps of a problem and helped the students visually see what they had to do. Since apps were interactive and provided instant feedback to students, they were more engaged with the content. In addition, iPads were very useful in providing individualized instruction based on student abilities, and this provided the most benefit to students with learning disabilities. Having the iPad during math class, students completed their tasks, and they took charge of learning. Therefore, use of iPad reduced distractions, eliminated behavior problems, and raised students' willingness to learn math. According to the comments from the teacher candidates, they noticed that using both interactive math apps and traditional teaching models allowed students to practice solving math problems in a fun way and increased their willingness to practice as well. From this study, researchers found that students were more involved with the content and had more opportunities for repetition and practice and were more focused when they had iPads (Kaur et al., 2017).

The authors concluded that iPads have the potential to help students with learning disabilities understand basic math content better when used in conjunction with traditional teaching methods. Besides, using appropriate math apps as a supplement tool was needed to help them practice particular learning skills (Kaur et al., 2017)

O'Malley et al. (2013) studied the effectiveness of using iPads to build basic math fluency for students with moderate to severe disabilities in special education setting. The investigation had two purposes; 1) how the iPad may assist students with disabilities in increasing basic math fluency rates, and 2) identify the advantages and barriers to using iPads to teach and learn (O'Malley et al., 2013).

The study was conducted for 4 weeks and ten 7th and 8th grade students (3 females, 7 males) with the following federal disability categories: autism, emotional disability, intellectual disability, multiple disabilities, other health impairments, specific learning disability, and traumatic brain injury participated in this study. Every child had an individual education plan (IEP) and got related services from special education. Basic math fluency was measured with timed math probes, and it consisted of 20 addition and subtraction problems. The baseline phase was conducted using a traditional instructional approach of paper and pencil assessment. The teachers timed and recorded participants' scores, once students completed the assessment. During intervention phase, the Math Racer application by i4software was used to practice and learn basic math skills on the iPad. The application in iPad timed and recorded the students' scores about 20 basic math problems. The dependent variable was the rate of basic math fluency gains, and the independent variable was the timed math probe in this study. The study analyzed data using dependent t-tests to compare the means between two related groups on the same continuous variable (e.g., pre- and post-test scores, baseline and intervention means).

Participants completed paper and pencil timed math assessments during the first week to determine baseline. During the second week, students practiced timed math probes through the iPad application. The intervention was stopped for week three and students completed paper and pencil timed math probes for a whole week. During the last week, the students practiced solving math problems using iPad app activities and data collection continued (O'Malley et al., 2013).

The results showed that teaching basic math using iPad was effective instruction for students with moderate to severe disabilities. First, teachers responded that the students appeared to be eager to participate with iPad activities. Moreover, the teachers were highly satisfied with the results and answered the intervention was completed successfully. When asked whether the intervention was worth it, 100% responded positively with either 4 (agree) or 5 (strongly agree) rate. In addition, the participants showed increased interest in content during the intervention, and their interest went down when they went back to baseline phases. Dependent t-test indicated two significant findings. First, the students were able to answer more problems correctly per minute during the iPad intervention phases ($M = 17.56,\ SD = 6.65$) than during traditional instruction phases ($M = 5.75,\ SD = 3.41$); $t(9) = -8.66,\ p < .001$. Second, the students significantly improved in solving speed (as measured by seconds). The time to complete math problems reduced from pre-test ($M = 1518.00,\ SD = 606.00$) to post-test ($M = 600.00,$ $SD = 375.95$); $t(9) = 7.09,\ p < .001$ (O'Malley et al., 2013).

In sum, iPads could be used as an effective and efficient instructional tool to teach basic math fluency for students with moderate to severe disabilities. Most of the students who participated in this study made fluency gains. The gap between baseline and intervention means was an average of 12 points in the number of correct problems per minute. Moreover, the teachers were strongly interested in expanding the use of iPads in their classroom instruction. It is a very important factor in the sustainability of an intervention. It depends not only on how well it worked in the classroom, but also on how well the educators who implement it recognize it (Fuchs & Fuchs, 2001).

Number Line Representation

I found three studies about the number line teaching strategy, and one of them introduced the effects of the number line teaching model and the other studies proved how it worked for students with disabilities.

Yilmaz et al. (2019) examined the usage of number line models with middle school students while they were solving real-life problems involving integers and they described the strategy to calculation using number line representation. The number line helps students to visualize the placement of numbers relative to each other, teachers to demonstrate calculations, and helps students develop visual strategies (Bramald, 2000; Heeffer, 2011; Murphy, 2011). In addition, the number lines strategy was a very familiar mathematics model to most students. And it was one kind of representational model which was widely applicable across concepts as well as grades (Gonsalves & Krawec, 2014).

Researchers in this study had 32 seventh grade students from a public middle school located in a low-income district in Ankara, Turkey as their participants in the study. They used convenience sampling, as known as “group of individuals who (conveniently) are available for study” (Fraenkel & Wallen, 2005, p. 99), for choosing their participants (Yilmaz et al., 2019).

Data used in this study were collected by questionnaire containing open-ended questions and semi-structured interview questions. In more detail, questionnaire items were open-ended questions and interview items were developed concerning the learning outcomes of Turkish middle-grade mathematics curriculum (MoNE, 2013). Moreover, the researchers also checked students’ solving process after students solved real-life problems to see if their calculation was correct or not. As an example, researchers examined the correctness of the arithmetical representation of the calculation to understand the relationship between students’ methods and mathematical expressions with the representation of calculation (e.g.: $-3 + 8 = 5$) (Yilmaz et al., 2019).

In this study, researchers introduced the concept images of number line models for real-life contexts. Depending on the contexts, different types of models were applied. Results revealed that students have three mental pictures of number line representations within real-life contexts: horizontal, vertical, and the combination of both number line representations, called the Cartesian plane. A summary of these results is presented in Table 2.

Table 2

Students' Number Line Images

Contexts	Typical models in real-life	Students' dominant images	Exceptional models
Thermometer	Vertical	Horizontal	
Movement	Horizontal	Horizontal	
Profit and Loss	No specific model	Horizontal	Cartesian plane model
Sea Level	Vertical	Vertical	
Historical Timeline	Horizontal	Horizontal	

The number line was used as a useful calculation tool and this strategy included a solution method that represented the calculation on the number line. Although students were not able to utilize the number line models to represent calculation, they used it as a calculation tool by equal partition and jumping method and necessary partition and jumping method. In equal partition and jumping method, students determined movements on the number lines. Students partitioned the number line in equal parts and moved by jumping on it. They made solutions by placing numbers on the line, leaving small equal parts between the numbers, and then using jumps between the numbers. Using this strategy, they solved the thermometer problem, the depth mentioned in the sea level problem, and the steps in the movement problem as well. According to the results of the students' calculations in their arithmetical sentences, students who got the correct answers of arithmetical sentences can use the number line for calculation with equal partition and jumping method successfully (Yilmaz et al., 2019).

Another method that represented the use of a number line model for calculation was the necessary partition and jumping method. In this strategy, students partitioned the number line in unequal parts. This strategy was necessary for large jumps among numbers in the calculation. Researchers examined how students used the necessary partition and jumping method and the corresponding arithmetic sentence they created. The results showed that students can utilize the number line for calculation integers. In other words, using the number line as a calculation tool helped students to correct their arithmetical errors.

In conclusion, a number line is a good educational tool to represent calculation (Gallardo & Romero, 1999; Van de Walle et al., 2010). This teaching strategy is used for not only addition and subtraction, but also multiplication and division when students learn calculation of integers (Van de Walle et al., 2010). Like the results of several previous studies (Klein et al., 1998; Resnick & Ford, 1981; Selter, 1998), in this study, students could make small movements or jumps using different strategies including counting ones, separating the number line in equal or unequal parts supporting to find the answer for numbers up to 100. This study added extra evidence on previous findings suggesting that students tended to put into action on number line models operations with integers as well as addition and subtraction with whole numbers. However, this study has some limitations. First, this study was conducted in one classroom in Turkey. That means there is a possibility to have different outcomes with students of different cultural and mathematical backgrounds and grades. Second, the results are based on limited data, which only included the real-life context of the problems and the questions asked to them during the interviews. Last, the interpretation of the results of this study could be changed by using different theoretical perspectives and methods. Nonetheless, the number line models give a visual aid for students to examine the relationships of integers with each other and help learning integers and operation of integers (Yilmaz et al., 2019).

Gonsalves and Krawec (2014) studied that number lines can be a model to solve word problems as part of a comprehensive problem-solving intervention to enhance the conceptual understanding of math word problems for students with learning disabilities. Researchers observed the class of Mrs. Wilson, a middle school math teacher and homeroom teacher of inclusive sixth grade. Ten students participated in this study and one student among them had learning disabilities, Maria. Before starting intervention, researchers introduced the instructions on using number lines to solve word problems. Instruction in visualizing strategy should be divided into two phases: (1) translating the problem to the number line, and (2) interpreting the number line representation (Skoumpourdi, 2010).

Step 1: Translating the Problem. Students with language or reading and math difficulties have a challenge to understand the problem's key features linguistically, such as comprehending the problem through a verbal translation of its important information. A number line representation can play the role of a bridge to connect the gap between linguistic understanding and mathematical understanding by illustrating the problem information. There are three important components to translate a word problem onto a number line: (1) the relevant information that was given in the problem, (2) the interrelationships or connections among that information, and (3) the goal or the question(s) in relation to the other information. Each component is equally important as a tool to plan to solve the problem when students use a number line representation. Thus, teachers should focus on these three components clearly in order that students fully understand how to identify the information from the problem, include it into the representation, and then translate that representation into a means of solving problem (Gonsalves & Krawec, 2014).

The first component is identifying the relevant information. Students should learn the way to extract the relevant information from the problem by paraphrasing the problem, such as by putting the problem in their own words. During this process, students can underline the important information for memory and recovering the information for the representations later. Thus, irrelevant information, which affects to have an inaccurate representation and leads to the wrong answers, should be removed by crossing out. Once students identify relevant information from the problem, they can start to construct the number line representation by determining what interrelationship exists between the pieces of information of the problem. To find the interrelationship provides the students with a framework for solving the problem and it is the hardest parts for students with LD to catch (Van Garderen, 2007). Finally, students must place the unknown of the problem on their number line representation. This step entails putting a question mark to indicate what they are looking for. After this, students are taught to check several times to ensure that the representation is accurate and informative before they move on to the next step (Gonsalves & Krawec, 2014).

Step 2: Interpreting the Number Line. Once students have an accurate number line model, they must examine and interpret this model to develop a plan to solve the problem. The representation completely generated by student is very meaningful and lessens the difficulty of the problem (Van Garderen, 2007). In this step, students must count how many steps are necessary to get the solution by checking the number of unknowns that are in the representation. Then, they should determine the appropriate operations as shown by the relationships on the representation. For instance, if a specific number of equal groups divided the number line and the unknown is the combined amount (or total) of all the groups, then students needed to catch that multiplication is the appropriate operation (Gonsalves & Krawec, 2014).

In this study contained a case study on Maria, a sixth-grade student with LD, was taught the use of number line representation method for finding of a single-subject research project (J. Krawec, in preparation) chiefly designed to assess the effectiveness of a modified math problem solving intervention (Montague, 2003). Maria was instructed to focus on paraphrasing and visualizing during her full intervention, such as instruction on reading, paraphrasing, visualizing, hypothesizing, estimating, computing, and checking. She had a trained research assistant assigned to her and the assistant conducted the intervention, consisting of a 35-minute session three times per week. After each session, the assistant assessed Maria's progress by four arithmetic word problems to check her understanding. Maria could progress to the next instruction once she showed at least 75% on 3 of 4 consecutive measures on her process. The assistant noted her progression of baseline, lesson 1, lesson 5, 2-month maintenance and 4-month maintenance (Gonsalves & Krawec, 2014).

The result of participants including Maria and the other nine students in Mrs. Wilson's 6th grade math class demonstrated increased accuracy and ability of schematic nature in their word-problem representation after just one lesson of number line. In addition, the majority of students mastered the skills after taking five lessons and they improved in their problem-solving scores after visualizing instructions. Therefore, the result of the case study validated the number line representation strategy's efficacy to grow ability of students with and without learning disabilities to solve math problem and it can be a flexible and concise tool that helps them with planning, carrying out, and solving math word problems. This instruction model will be a very supportive tool to develop proficiency in math word problem solving for students with LD, especially when teachers teach the content, sequence, and approach of math problem-solving instruction. The researchers concluded that understanding to construct and interpret number lines will facilitate the growth of more sophisticated schematic diagrams to solve more advanced problems (Gonsalves & Krawec, 2014).

Koç (2019) examined the impact number line and Educreations had on second-grade students' verbal and written ability in solving 3-digit addition and subtraction operations using whole numbers. This present study is a collaborative action research project, and it examined the impact of the collaboration of number line and Educreations (Koç, 2019).

Educreations (http://www.educreations.com) is an educational application that provides various virtual tools including an interactive whiteboard and screencast tool which users can add videos, voice-overs, images, and annotations to instructional presentations to help conceptional understanding as an additional explanation. Students can comment on or draw on the whiteboard and both teachers and students have variety packs of ink color to use. In addition, this app is easy to use and allows both teachers and students to create videos, presentations, and craft (Johns et al., 2017, p. 56). Educreations can be used in not only math lessons, but also science lessons to show the steps to solve for solving math problems and create explanation videos for science experiments (Johns et al., 2017).

There is a lot of research about the effect of the number line strategy. For instance, Woods et al. (2018) mentioned, "A number line is a visual representation that illustrates the order and magnitude of numbers" (p. 230). They also concluded that it "point[s] to the value of using visual representation of mathematics concepts for supporting the development of students' mathematics understanding". Previous studies (e.g., De Smedt et al., 2009; Siegler & Opfer, 2003) revealed that the performance of arithmetic tasks using number lines from kindergarten and first grade students was related to achievement in grade 1 and grade 2, respectively. Consequently, Frykholm (2010) concluded that using the number line helped students "develop greater flexibility in mental arithmetic as they actively construct mathematical meaning, number sense, and understandings of number relationships" (p. 4).

This project contained pre-and post-assessments to examine how the students performed on the given 3-digit addition and subtraction problems and all assessments were created by traditional paper-and-pencil method. Thirty-two second-grade students (ages 7-9) and one classroom teacher participated. This project took place at a Midwestern urban school and the school administration supported and encouraged the usage of educational technology for teachers in their classroom. Several number line apps were considered to support student learning and practice addition and subtraction in this action project. Participants and action researcher settled on the Educreations app as a tool for student practice with the number line, addition, and subtraction. They concluded that Educreations included various features that provide benefit to students and implement classroom activities they had planned (Koç, 2019).

When students took a test using the Educreations app, they were instructed to make the number line and write on the app's whiteboard while they explained how to get the answers and methods. Students also could watch a video at the same time, and the length of video was not too long; around one minute. The purposes of this video were as a formative classroom assessment tool, reflection tool for this study, and conceptual learning for the students by self-monitoring. Besides this, the teacher could check students' understanding and how they solved given addition and subtraction problems using Educreations while watching the videos. At the end of the classroom activities, the students were required to answer about the value of using the number line with three-digit problems through a Google form questionnaire (Koç, 2019).

The action research project concluded that teaching through the number line as an instructional strategy along with the students' self-made video by Educreations enhanced student knowledge and basic mathematics problem-solving skills. The result of pre-assessments, the students did not explain and even write how they solved the given problems. On the other hand, the results of post-assessment showed that all students got the right answer on the addition and subtraction, and they explained the process correctly. Compared to addition and subtraction, the students did better on addition than subtraction problems. Regarding the students' thoughts of using the number line, the students responded that the number line helped them understand about place value and the reasons for using the number line in addition and subtraction operations. The students did not much use the number line when they were taking traditional paper-and-pencil assessments, however, they frequently drew and marked on the number line when they solved addition and subtraction problems through Educreations. The videos in Educreations allowed the students to precisely put into practice visualizing the number line, the place value, and how math problems can be solved. Through this result, the number line and Educreations together can help students with and without LD improve and develop a strong foundation for their conceptual understanding of the number system, place value, addition and subtraction, and early algebra (Koç, 2019).

Concrete-to-Representational Abstract (CRA) Framework

Many students with LD may struggle at the beginning point with the abstractness of the number line representation and so may benefit from the instruction of concrete-to-representational-abstract (CRA) sequence (Mancl et al., 2012; Mercer & Miller, 1992). Before the CRA sequence, there was Bruner's (1966) stages that explained how students use representations to make conceptual understanding (Milton et al., 2019). Bruner's stages involved three stages, and the first one was inactive stage. It contained the use of objects without internal representation. The next was iconic stage, where children develop mental images of what they have constructed and can visualize concepts in their mind. After that, the symbolic stage came as the last stage, where the representation information can be saved in specific form of symbols that can be organized and classified (Milton et al., 2019).

Witzel et al. (2008) examined implementing CRA with secondary students with learning disabilities in mathematics. The CRA instructional sequence contained three levels of learning. The three levels of the CRA instructional sequence included the following.

1. C: Learning through *concrete* or hands-on instruction using actual manipulative objects.
2. R: Learning through pictorial *representations* of the previously used manipulative objects during concrete instruction.
3. A: Learning through *abstract* notation such as Arabic numerals and operational symbols.

The connections within and between the CRA were critical for students to demonstrate proficiency. CRA was an effective instructional process for students to teach the procedures of mathematics and essential mathematical concepts, which helped students become mathematically proficient (Witzel et al., 2008).

There were not many research-supported mathematics interventions for students at risk and for students with learning disabilities (Stein et al., 2006). Some interventions that have proven beneficial for students with learning disabilities were mnemonic instructional strategies (e.g., Mastropieri & Scruggs, 2007) and CRA instructional sequence (e.g., Maccini & Hughes, 2000). Even though research evidence is available to exhort teachers to use the CRA instruction sequence in their secondary mathematics classes, few middle and high school textbooks adequately and correctly use the CRA instructional sequence. Therefore, the mnemonic CRAMATH was invented to help guide teachers in the design of effective mathematics instruction based on the CRA instructional sequence (Witzel et al., 2008).

The purpose of the CRAMATH strategy is to help to guide teachers' instructional planning to involve the components of CRA instruction in their lessons. CRAMATH was consisted of seven steps to implement CRA successfully for math instructors in secondary math skills.

1. C: Choose the math topic to be taught.
2. R: Review procedures to solve the problem.
3. A: Adjust the steps to eliminate notation or calculation tricks.
4. M: Match the abstract steps with an appropriate concrete manipulative.
5. A: Arrange concrete and representational lessons.
6. T: Teach each concrete, representational, and abstract lesson to student mastery.
7. H: Help students generalize what they learn through word problems

1. *Choose the math topic to be taught*: It is important to know what math topic will be taught and how it will be taught before the teacher introduces a new math concept to students. Any math topic can be examined by the teacher for possible development of CRA instructional steps. Currently, many school districts try to design pacing instruction guides and calendars of what content to teach each week or even each day (Witzel et al., 2008).

2. *Review procedures to solve the problem*: In the second step, review procedures to solve the problem, the teacher's role is important. The teacher will decide the objective of the instructional lesson. For example, when the topic of middle school standard is solving for unknowns using one-and two-step equations, the teacher would make a list of the desired steps to solve the problem: identify the variable; subtract, add, multiply, and divide to leave alone the variable and coefficient; arrange equations to balance both sides of the equal sign; let the calculations begin, keeping in mind the order of operations; continue all balanced calculations until the coefficient is 1; total both sides of the equation to determine the answers; and evaluate the answer (Witzel et al., 2008).

3. *Adjust the steps to eliminate notation or calculation tricks*: Once the steps are sketched, the teacher should go over each step and remove any tricks or shortcuts that are present. In this stage, teaching one general approach that solves multiple problems will be a more efficient process for the students who have difficulties with memorization. They may improve shortcut strategies as they learn a skill, but the use of those strategies should be monitored for accuracy and generalizability across the concept from one skill to the next (Witzel et al., 2008).

4. *Match the abstract steps with an appropriate concrete manipulative*: In this stage, students will combine their conceptual understanding and their interactions with concrete

objects. Therefore, to choose concrete objects that cover multiple skills under the same conceptual skills is important to increase generalizability of math rules, procedures, and concepts (Witzel et al., 2008).

5. *Arrange concrete and representational lessons*: Once the teacher prepares the concrete instruction to teach procedures, pictorial representations must be created which mimic the concrete manipulation. For instance, the teacher can teach subtraction of integers by removing sticks. Pictorially, the teacher uses sticks to represent for negative and positive numbers by putting both symbolic signs in front of the set of sticks. The teacher shows by placing an “X” over the sticks subtracted (Witzel et al., 2008).

6. *Teach each concrete, representational, and abstract lesson to student mastery*: Each stage of learning (i.e., concrete, representational, and abstract) is practiced to mastery to retain the students’ math skills. To help students transition from one stage to the next one, accurate and consistent assessment is necessary. Especially using clear, consistent, and appropriate language and terminology which explains mathematics principles of teachers helps transition from the representational stage to the abstract stage (Witzel et al., 2008).

7. *Help students generalize what they learn through word problems*: In general, most students with disabilities have low ability to apply math concepts and skills into word problems without any explicit guidance. Therefore, teachers should prepare the word problems that involve relevant real-life problem, while the students work to learn the concepts and computation skills. By solving incorporating word problems, teachers can teach the social relevance, importance of learning math skills, and set up the motivations which are often lacking in math lessons.

Moreover, the language experience embedded during instruction at each stage help students improve in problem-solving situations such as word problems (Witzel et al., 2008).

In summary, implementing the explicit CRA instructional sequence helped in the understanding of math concepts for students with disabilities, and it can be applied across various mathematical concepts as well. The CRA instructional sequence can be flexibly designed for students with and without disabilities by special education teachers or general education teachers for inclusion classroom. In any situation, it is important to examine and implement research-validated instructional models that can support students with mathematical difficulties and LD (Witzel et al., 2008).

Milton et al. (2019) conducted a quantitative and qualitative study that examined the effects of alternating CRA multiplication and division instruction on students' conceptual understanding and their mastery of unknown facts. For this study, five 4th through 6th-grade students with learning disabilities who failed to master the multiplication facts participated. Five students met the following criteria: (a) have permission from their parents to participate, (b) prove a score of 20% or less from the test of multiplication and division computation, and (c) receive special education services. The students in this study were Tyler, Julia, Wyatt, Sam, and Antoni. Among five participants, only Tyler was eligible under the category of other health impairments (OHI) because his medical condition affected attention and his educational performance. The others were eligible under the category of specific learning disabilities (SLD). All students took a math class in general education and received special education services in a resource setting for intervention support in mathematics. Since this research was both quantitative and qualitative, the researchers collected assessment probes and interviews from teachers respectively as data. The researchers made three types of assessment probes, including 1-min multiplication and division tests and untimed division tests for quantitative data. Division was the target skill, so they assessed students' acquisition through untimed probes. To collect qualitative data, they involved an interview in which the teachers asked students the way to solve the problems, verbally describe, draw, and use another operation to check their conceptual understanding (Milton et al., 2019).

Prior to instruction, to check the students' developed conceptual understanding of multiplication and division, and the relation between the two operations, the instruction alternated between the two operations. The teacher taught multiplication during the first lesson and the following lesson was the first division lesson using manipulative items to teach both operations. The third instruction was multiplication and the fourth was division. Students can move from one lesson to the next lesson once they completed independent practice problems with at least 80% accuracy. When students could not pass the independent practice problems, the teacher repeated the lesson (Milton et al., 2019).

During instruction, the lessons consisted of the CRA instructional sequence and the teacher presented lessons using explicit instruction. The first three multiplication and first three division lessons were concrete. The instruction of concrete multiplication contained translating a problem, such as 6×3 into words. Students should understand the units within six were sets or groups. The three meant the copies or same amount of each set or group, and therefore, the students finally understood 6×3 as six sets of three. The teacher taught students to place six plates on the table to represent sets and three base-ten blocks on each plate. Students learned division within two different approaches: quotative and partitive approach. When students learned partitive division, the instruction involved translating a problem, 18 ÷ 3, into words; 18 can be made into how many equal sets of three? Once the first multiplication and division lessons were finished, the teacher taught and emphasized the relation between the two operations during modeling and practicing (Milton et al., 2019).

Students learned to solve problems using drawings and pictures at the representational stage. At this stage, students were given learning sheets with preprinted drawing lines for practice, and then the teacher and students had to draw their own models using horizontal lines and vertical tallies. For multiplication, they made a horizontal line for each group and put short vertical tallies on the line to show the number of objects in each group. For division, they made a large group of short tallies and marked smaller groups of the same size to decide the number of groups. Once students completed taking instruction at the representational level, they learned a strategy to help in attending to key features of multiplication and division problems: **d**iscover the sign, **r**ead the problem, **a**nswer, or draw and check, and **w**rite the answer (DRAW; Mercer & Miller, 2010a, b). The DRAW strategy made students attend to detail while solving problem such as attention to the sign and numbers, a reminder that facts can be drawn if not memorized, and a prompt to write the answer (Milton et al., 2019).

At the abstract level, students committed the DRAW strategy to solve problems. The strategy allowed students to attempt to answer from memory and let them draw if needed. If students completed all 10 lessons before achieving mastery including accuracy (three consecutive probes with 10/10 correct) or fluency (three consecutive probes with 30 correct digits written in 1 min), the teacher continued to teach abstract lessons and provided fluency activities like games with flash cards that authors of the instructional manuals suggested (Milton et al., 2019).

During intervention, the researchers noted students' performance using graphs. The graphs displayed students' baseline, division accuracy, and fluency in division and multiplication. The researchers analyzed the data visually, noting the range, level, immediacy of effect, the number of probes to criteria for mastery, and the percentage of non-overlapping data points (PND) between baseline and intervention (Scruggs & Mastropieri, 2013). They calculated effect size using the Tau-*U* statistic to show a trend in a student's multiplication fluency baseline. Three researchers analyzed the qualitative data, such as noting students' language and behaviors related to the understanding of division and multiplication (Milton et al., 2019).

Through the study, the researchers demonstrated a functional relation between the use of CRA instruction and the students' accuracy and fluency in multiplication and division facts. All five participants achieved the accuracy criterion in division and fluency criterion of answering 30 correct digits in 1 minute on three consecutive multiplication and division probes with 100% accuracy. Even after instruction, the students maintained their skills and their confidence in mathematics increased as well. The researchers made an inference regarding the students' confidence using reports from special education and general education teachers. The teachers noticed that the students' participation increased through verbal contributions during whole-class discussions. Also, the teacher found that students' positive comments about mathematics problems related to multiplication and division increased. Before CRA instruction started, all five students could not write more than four correct digits on the division problems. However, after the intervention, they exceeded the criterion of 30 correct digits. In addition, the students explained the way to solve the problems using words and drawing. They used few words and did not provide more than a restatement of the equation before CRA instruction. After intervention, students answered using more words, sentences, and drawings to prove how they understood division (Milton et al., 2019).

In conclusion, the CRA instruction sequence demonstrated success when applied in a natural setting for students with SLD and OHI, a part of their regularly scheduled supplemental mathematics instruction. Implementing CRA instruction not only improved students' multiplication and division facts, but also developed students' explanations. The results related showed that CRA instruction might better emphasize concepts than remedial instruction, in which educators teach operation separately. Therefore, this alternating sequence could be an efficient strategy to teach multiplication and division to students with learning disabilities in need of remediation (Milton et al., 2019).

Mancl et al. (2012) extended that the previous literature related to the use of CRA with integrated strategies (Harris et al., 1995; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Mercer & Miller, 1992; Miller & Kaffar, 2011; Morin & Miller, 1998) to the ability of subtraction with regrouping and to specifically duplicate the instructional procedures of Ferreira (2009) with a few modifications. Five 4th and 5th grade students who were Jorge, Betty, Amy, Julio, and Harry participated in this study and all students were identified as having a learning disability in mathematics. They all received special education services, such as 30 minutes of Tier-3 mathematics intervention from a special education teacher in a resource classroom (Mancl et al., 2012).

The researchers used two measures: baseline probes and intervention probes. Both probes consisted of eight computation problems and two-word problems. The problems of baseline probes had different levels and types of computation, such as (1) four computation problems that included 2-digit numbers and only one of the four included a zero, (2) four computation problems that included 3-digit numbers and only two of the four included a zero, and (3) two paragraph format word problems that included 2-digit numbers. These problem sets were guided in a previous study (Ferreira, 2009) and the result of baseline probes of the five students indicated that they were similar in difficulty level. On the one hand, the problems in intervention probes were same type of problems that students learned during the lesson, such as regrouping from hundreds, regrouping from tens. The intervention probes aligned with the lesson contents and were also guided by previous study (Ferreira, 2009).

The intervention consisted of 11 lessons with 30-minute of each. For this intervention, five instructional materials were needed: scripted lessons, learning sheets, manipulative devices, place-value mats, and student notebooks. In total, 11 scripted lessons and learning sheets were used to teach subtraction with regrouping instruction. Each script contained what the teacher was to say and do during the lesson. Thus, each lesson scripted had five sequential lesson components: advanced organizer, describe and model, guided practice, independent practice, and problem-solving practice. The 11 learning sheets included 10 learning sheets with 14 problems and one learning sheet with the list of strategy steps. Three-dimensional plastic base-10 blocks were used as a manipulative device during five lessons out of 11. The size of place value mats was 11 × 18 and the mats were made of laminated construction paper. Each mat had three columns for the place value of ones, tens, and hundreds. The mats were used with base-10 blocks to indicate and solve subtraction with regrouping problems. Each student had one folder to organize and store their materials, including learning sheets. Students were given the learning sheets with their progress chart on the top of the sheets, so they could check it whenever they opened their notebooks. The chart showed the students' learning sheet scores as a graph, and it had been a motivator for the students (Mancl et al., 2012).

The instructional routine was filled with 11 scripted intervention lessons (Miller & Kaffar, 2010). Each lesson was proceeded by CRA strategy and contained both computation and word problems. The first five lessons were concrete level instruction. During the concrete level instruction, students used base-10 blocks, place-value mats, and different learning sheets that were given each lesson. The teacher reviewed the concept of 2-digit subtraction without regrouping during the first lesson. For the next lesson, the teacher reviewed the concept of place value of tens and ones. Thus, the teacher introduced the students to solving 2-digit numbers of subtraction problems using regrouping a ten to ten ones. The teacher focused on students' understanding and the ability to solve 2-digit number subtraction problems using regrouping in Lesson 3. The third lesson also involved the introduction of BBB Sentence (i.e., **B**igger number on **B**ottom means **B**reak down and trade.) (Mercer & Miller, 1994) to help students when regrouping is necessary. Lesson 4 reviewed the expansion of the place value concept up to hundreds and introduced solving subtraction problems of three-digit number required regrouping to students. On the last lesson, Lesson 5, the teacher emphasized the students' understanding of subtraction related to regrouping when the problems had 2- or 3-digit numbers (Mancl et al.,2012).

The representational level instruction was proceeded on the next three lessons, Lesson 6, 7, and 8. During this stage, students learned drawing of base-ten blocks instead of using the physical blocks. Lesson 6 involved practice to draw base-10 blocks that represented the first number of the problem and marks on the top of the problems drawing, such as squiggly lines representing regrouping and straight lines representing taking away. Students practiced drawing two-dimensional drawings of base-ten blocks when they solved subtraction problems that involved 3-digit numbers and required regrouping during Lesson 7. They also practiced drawing during Lesson 8, but they solved not only 3-digit numbers, but also 2-digit numbers or 3-digit and 2-digit numbers (Mancl et al., 2012).

Before moving on to the abstract level, one lesson, Lesson 9, the strategy lesson, was required. At this stage, the teacher taught the specific strategy to help students gain fluency related to memorizing the steps used to solve 2- and 3-digit subtraction problems without using or drawing base-10 blocks. Lesson 9 involved the steps to the mnemonic device RENAME (Miller et al., 2011). The RENAME strategy means (1) **R**ead the problem, (2) **E**xamine the ones column: Use BBB Sentence for ones, (3) **N**ote ones in the ones column, (4) **A**ddress the tens column: Use BBB Sentence for tens, (5) **M**ark tens in the tens column, and (6) **E**xamine and note hundreds: exit with the quick check (Mancl et al., 2012).

Abstract level instruction was taught during the last two lessons, Lessons 10 and 11. Both lessons included the subtraction problems with number symbols only. The students applied the RENAME strategy to solve the problems. Lesson 10 involved only 2-digit problems and Lesson 11 involved 3-digit number problems, respectively. The problems were designed to use the RENAME process to find the answers without the use of manipulative devices or drawings.

During the last lesson, the teacher reinforced the importance of developing fluency about solving regrouping problems (Mancl et al., 2012).

All five students completed the 11 intervention lessons from their special education teacher in a resource room setting. They finished their learning sheets, which were created through the five lesson components (i.e., advanced practice, describe and model, guided practice, advanced practice, and problem-solving practice). The feedback was provided to students and the percentage of score was plotted on the progress chart and shown as a graph, which was included in each student's notebook (Mancl et al., 2012).

The results of this intervention were very positive. Table 3 illustrates all five participants' means, standard deviations, and percentage point gains.

Table 3

Mean Percentage Scores, Standard Deviations, and Percentage Point Gain Scores

Participant	*Baseline M/SD*	*Intervention M/SD*	*Percentage Point Gain Score*
Jorge	13.33/23.09	89.09/12.21	75.76
Betty	0/0	92.73/9.05	92.73
Amy	0/0	90.00/10.00	90.00
Julio	0/0	92.00/6.32	92.00
Harry	0/0	75.71/21.02	75.71

Although some participants had to repeat the lesson, all participants achieved mastery (80% or higher) on all 11 lessons. Only one participant, Julio, did not repeat any lesson, three participants (Jorge, Betty, and Amy) repeated one lesson, and Harry had to repeat three lessons. The results of this study revealed that using CRA instructional sequence with integrated strategy was beneficial and explicit instruction to teach to solve the multi-digit subtraction problems that required regrouping for upper elementary school students with LD in mathematics. Moreover, the findings demonstrated the previous research that the benefits of CRA sequence for various basic mathematics skills (e.g., basic addition, multiplication, and division facts; addition with regrouping; fraction; algebra) (Butler et al., 2003; Harris et al., 1995; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Miller & Kaffar, 2011; Miller & Mercer, 1993; Witzel et al., 2003). The findings also concur with the findings that the benefits of CRA sequence for teaching subtraction with regrouping by Flores (2009, 2010), and Ferreira (2009).

Gibbs et al. (2018) examined the use of CRA to teach multiplication without counting for students with disabilities and mathematics difficulties. 15 third- and fourth-grade students who received special education services in mathematics in the general education classroom participated in this study. The students received supplemental mathematics instruction twice a week for 30 minutes in the resource room according to their individualized educational programs (IEP). The worksheets used in supplemental instruction did not include CRA, but it showed representations of numbers using pictures of base-ten blocks and numbers only. The primary researcher, a special education teacher, conducted 11 CRA instructions to replace supplemental math instruction. The CRA intervention was implemented in small groups of three to six students. Students also received core instruction in the general education setting. They learned to draw out representations of numbers, then solving problems using numbers only in the general education classroom (Gibbs et al., 2018).

The researchers collected the data of two assessments to gather information on the students' ability to solve multiplication facts with digits 1 to 10 and skip counting before and after CRA intervention. For multiplication assessments, students should complete 50 one-digit facts for 2 minutes and they should finish the skip counting assessment without time limit. This study used a paired-samples *t* test to analyze differences in pretest and posttest measures of students' multiplication ability without counting (Gibbs et al., 2018).

All 11 lessons involved explicit CRA instruction using skip counting as a strategy. Each lesson included solving multiplication problems using the strategy they had learned by number 2 through 9. Before they started intervention, the teacher reviewed the concept of multiplication as repeated addition. The teacher explained flexible ways of counting to help students remember and solve multiplication problems. Teacher and students played a game of counting dots on a card without touching and counting each dot. For example, in the beginning, the students would touch the dots counting by ones instead of recognizing groups of numbers. Their special education teacher let them count by ones, but then had students turn back to count by recognizing groupings of dots. Eventually, students freely recognized groupings of the dots skip counting by ones. This game later turned into a flashcard review after lesson three. The teacher also taught solving multiplication facts through putting base-10 blocks in groups, drawing tally marks in groups, or using numbers only. Once teaching the concept of multiplication using base-10 blocks, drawing tally marks, or numbers only, the teacher conducted guided practice. The teacher let them solve multiplication problems using the three strategies. After the guided practice, students practiced solving multiplication problems using base-10 blocks, drawings, and numbers without counting independently. The teacher provided feedback, but did not give answers, so students needed to find the errors and correct answers by themselves. Through this activity, the students solved problems and made corrections independently and the teacher assisted them as needed (Gibbs et al., 2018).

The results from the *t* test indicated a significant difference in before and after assessments scores for multiplication, $M = 13.27, SD = 11.85, t(17) = -4.34, p = .001$. The findings of this study advocated previous research about the CRA instruction as effective instruction when teaching students with disabilities or mathematics difficulties (Flores et al., 2014; Miller, 2009; NMAP, 2008). The students showed increased ability to count and view numbers in flexible ways after CRA intervention (Hinton et al., 2016). Moreover, they demonstrated a better understanding of quantities when solving multiplication problems. After CRA instruction, students could count starting with a multiplier or multiplication based on which number was more efficient counting for them. Also, they did not need to couch each object or tally mark to count. In other words, students used counting in a flexible way as a strategy to multiply after the intervention. In conclusion, this study verified how to implement effective and explicit CRA instruction to teach multiplication in conjunction with the strategies which build off of skip-counting to improve numerical sense for students with disabilities (Gibbs et al., 2018).

Table 4

Summary of Chapter II Findings

Author(s)	Study Design	Participants	Procedure	Findings
Burns, Kanive, & DeGrande (2012)	Quantitative	145 third graders and 86 fourth graders participated.	The intervention proceeded at least three times per week for 8 to 15 weeks. Participants spent approximately 5 to 15 minutes working independently on solving math problems each session. Each session contained 40 math problems, which were presented on computer screens in large font with three potential answers.	Computer-based math fluency (CBMF) intervention was effective in increasing math skills of students identified as at risk for math difficulties.

Table 4 (continued)

Bouck, Joshi, & Johnson (2013)	Quantitative	146 sixth grade and 149 seventh-grade students. The students with disabilities were included in both grades and they were categorized as learning disabilities, hearing impairments, and ADHD.	Participants completed assessments aligned to state standards, and they used a calculator to solve problems. The problems consisted of multiple choices questions and open-ended problems. Students completed 16 assessments for 9 months in one school year. They indicated "c" for the problems solved using calculator. (self-reported data) All students had 10 minutes to finish each assessment.	Students with and without disabilities got more correct answers when they used a calculator.
Kaur, Koval, & Chaney (2017)	Qualitative	Ten students: three African American females, four African American males, and three Caucasian males. Tutors were ten different teacher candidates.	Ten teacher candidates provided one-on-one math tutoring services to ten students for five weeks. Teachers used ten free apps to address math standards and topics provided by the classroom teacher. They reflected on their tutoring experience at the end of each week about the effectiveness of apps and any changes that they had to make for the following week.	Students with learning disabilities much improved in conceptual understanding of numbers, order of operations, expressions, and multiplication and division skills when adding iPad to traditional teaching methods.
O'Malley, Jenkins, Wesley, Donehower, Rabuck, & Lewis (2013)	Quantitative	Ten 7th and 8th grade students (3 females, 7 males) with disabilities	The study measured timed math probes involving 20 addition and subtraction problems. The intervention was conducted for four weeks, and the participants completed math probes using iPad activities during intervention.	iPad could be used as an effective and efficient instructional tool to increase basic math fluency for students with moderate to severe disabilities.
Yilmaz, Akyus, & Stephan (2019)	Quantitative	Thirty-two 7th grade students	Participants took a test and the test consisted of open-ended questions and semi structured interview questions. All the questions were related real-life context. There were five open-ended questions and students required to represent the given information as an arithmetic equation for each question. With the given empty horizontal and vertical number line models, students had an opportunity to choose which number line model they will use to solve the problem.	Number line provided a visual aid for students and helped them learn integers and operation of integers.

Table 4 (continued)

Gonsalves & Krawec (2014)	Qualitative	Mrs. Wilson's inclusive sixth grade class students. Case study: Maria, a sixth-grade student with LD.	Step1) Translating the Problem 1. Identifying the Relevant Information 2. Showing Relationships among Problem Parts 3. Including the Problem Questions Step2) Interpreting the Number Line A trained research assistant conducted the sessions with Maria during a 35-minute homeroom period three times a week. The session progressed for five lessons and maintained 2-and 4-month time.	Participants improved the accuracy and the schematic nature of their word-problem representations after just one instructional session in this skill. Also, most of them realized their improvement in problem-solving scores after the visualizing lessons.
Koç (2019)	Quantitative	Thirty-two Second-grade students (age 7-9)	The project was implemented with number line for teaching 3-digit addition and subtraction operations to participants. The classroom teacher used Educreation program and number line to teach the lesson. The project included the pre- and post-assessment and written explanation of solving problems. Both assessments were used in paper-and-pencil method.	Using number line as an instructional strategy to teach addition and subtraction increased student's conceptual understanding of place values and their ability to solve operations with three-digit problems.
Milton, Flores, Moore, Taylor, & Burton (2019)	Quantitative & Qualitative	Five students who received special education services and their score of untimed assessments of multiplication and division computation was 20% or less. Tyler was eligible under the category of other health impairments (OHI) and the other students (Julia, Wyatt, Sam, and Antoni) were eligible under the category of specific learning disabilities (SLD).	During intervention period, the students were taught multiplication and division using CRA instruction. They could move to the next level once they passed the test and they learned DRAW strategy at the abstract level. The intervention proceeded for 10 weeks, and the data was collected for quantitative and qualitative.	All five students achieved the goal of multiplication and division facts with 100% accuracy. They could use DRAW strategy to solve problems and their confidence in math was increased after CRA instruction.

Table 4 (continued)

Mancl, Miller, & Kennedy (2012)	Quantitative	Fourth and fifth graders five students with LD in mathematics.	11 lessons with 30 minutes each. The intervention was based on CRA instruction and RENAME strategy was taught. The students learned from one education teacher in resource room.	All students achieved higher scores right after the intervention started. Their math skills of subtraction with regrouping increased and confidence in math also increased through CRA instruction.
Gibbs, Hinton, & Flores (2018)	Quantitative	15 third- and fourth-grade boys and girls with disabilities who received special education services in mathematics.	All students received supplemental math instruction in the resource room two times a week for 30 minutes according to their IEP. The supplemental instruction did not include CRA, but one primary researcher, a special education teacher, implemented CRA intervention to replace supplemental instruction. The CRA instruction was implemented in small groups of three to six students.	Results indicated that the CRA instruction was an explicit and effective strategy when teachers teach multiplication to students with math difficulties. Also, the students increased in their ability to count and view numbers in flexible ways.

Chapter III: Conclusions and Recommendations

The purpose of this paper was to find effective interventions to teach basic mathematics skills for students with learning disabilities. Chapter I featured the background information of this topic and Chapter II presented a review of the related studies. In this chapter, I discuss the conclusions, recommendations for future research, and implications for current practices.

Conclusions

All of the 10 studies reviewed in Chapter II were related to visual-spatial representation methods that help students remember and memorize ways to solve math problems effectively. Four studies demonstrated the benefits of using technology devices: computer-based math fluency (CBMF) (Burns et al., 2012), calculator (Bouck et al., 2013), and iPad (Kaur et al., 2017; O'Malley et al., 2013). Another three studies proved the advantage of using number lines (Gonsalves & Krawec, 2014; Koç, 2019; Yilmaz et al., 2019) to teach and the last three studies described the instruction of concrete-to-representational-abstract (CRA) sequence (Gibbs et al., 2018; Mancl et al., 2012; Milton et al., 2019). In this section, I summarize and synthesize the research to reflect each section.

Technology Models

The effective interventions where teachers teach basic math skills to students with learning disabilities using technology devices included computer-based math fluency (CBMF), calculator, and iPad. Burns et al. (2012) demonstrated that a CBMF intervention was effective in increasing math skills of students with LD or identified as at risk for math difficulties. They found that increased repetition led to enhanced recall and fluency of the skill (Burns, 2005; Daly et al., 2000; Szadokierski & Burns, 2008) and CBMF encouraged students to practice independently. Bouck et al. (2013) suggested the use of a calculator during math classes for students with learning disabilities. Their data showed that students with disabilities answered more problems correctly when they used a calculator. Thus, they were more likely to answer questions correctly with a calculator as a tool. In this study, students used a calculator just for solving problems, but the researchers expected the students could learn how a calculator was used (e.g., checking answers, trying strategies, or reducing the cognitive load associated with mental mathematics) once they were provided explicit instructions of calculator use by teachers. Kaur et al. (2017) confirmed that using an iPad as a supplemental tool increased students' conceptual understanding of not only basic math abilities, but also numbers, order of operations, and expressions. The study found that iPads were helpful with activities, showed step-by-step procedures on many apps, and helped the students pinpoint the exact step where they missed or got confused. O'Malley et al. (2013) proved that the use of iPads could be an effective instructional tool to foster basic math fluency of students with disabilities. Furthermore, the students' participation and interest in the content improved when they learned using iPads.

Number Line

The number line strategy is widely used in the study of mathematical cognition, learning, and development (Schneider et al., 2018). All three studies about the number line strategy demonstrated that using a number line as an instructional strategy increased students' conceptual understanding of basic math and their ability to solve context problems. Students mostly used horizontal number lines rather than vertical ones when they solved context problems and just calculation problems. Yilmaz et al. (2019) found that students utilized the number line as a calculation tool by two different methods: equal partition and jumping method and necessary partition and jumping method. Students could solve problems by applying each method in appropriate problems. Gonsalves and Krawec (2014) introduced the steps to solve problems using a number line and showed an example of student-generated number lines reflecting four basic operations (addition, subtraction, multiplication, and division). They concluded that students (particularly those with LD) will use number lines as a supportive tool to improve proficiency in solving math word problems when teachers apply instructional content, sequence, and approach of math problem solving instruction in their math lessons. Unlike two other studies, the research of Koç (2019) suggested to use a number line and Educreations together to teach math to students with and without disabilities. The author concluded that using both methods together is very effective in laying the foundation for students' conceptual understanding of the number system, four basic math operations, and even expand to algebra.

Concrete-Representation-Abstract (CRA)

Several studies existed about CRA instruction and the findings from the previous studies concluded that it helped students do operations and problem-solving from basic operations (Harris et al., 1995; Morin & Miller, 1998; Peterson et al., 1988) to regrouping (Flores, 2010; Flores et al., 2014; Mancl et al., 2012) and expanded to algebraic concepts as well (Watt et al., 2014; Witzel et al., 2003). Interestingly, of the four studies reviewed, three studies involved specific strategy before moving to the abstract stage (Mancl et al., 2012; Milton et al., 2019; Witzel et al., 2008). Witzel et al. (2008) introduced the CRAMATH strategy, Milton et al. (2019) provided the DRAW strategy, and Mancl et al. (2012) applied the RENAME strategy during the intervention. Even though all three strategies used different names, all of them helped students memorize and solve problems easily. The common purpose of the strategies was to support students in memorizing the steps to solve problems and checking if their answers are correct or not before writing their answers. Moreover, all four studies proved that teaching basic math using the CRA instructional sequence was successful when implemented in a portion of the regularly scheduled supplemental math classes for students with disabilities. This demonstrated that CRA instruction can be one of the most effective strategies for students with LD when they are learning basic math.

Recommendations for Future Research

Present findings contributed to proving the effectiveness of the interventions to teaching basic math skills to students with learning disabilities. Through the studies being examined, however, there are some limitations and suggestions for future studies.

Most of the studies cited small sample sizes as a limitation. Five of the studies listed that they had a small number of participants and warned against the generalization of results to a larger population (Burns et al, 2012; Gibbs et al., 2018; Kaur et al., 2017; Milton et al., 2019; Yilmaz et al., 2019). Thus, Yilmaz et al. (2019) said that different outcomes could have been obtained with students of different cultural and mathematical backgrounds, class participants, and grades. Hence, future research should address this issue with studies that include larger groups and more rigorous experimental design.

Three of the studies lacked the fidelity of collected data (Bouck et al., 2013; Burns et al., 2012; Kaur et al., 2017). Kaur et al. (2017) used data from the perspectives of pre-service teachers only. Therefore, they recommended that the studies involved in-service teachers to better identify the benefits of using iPads with special needs students as future research. Bouck et al. (2013) used data rely primarily on self-reported data (i.e., calculator use). Burns et al. (2012)

collected data from a pre-post measure while the rest of the intervention systems monitored student progress. Thus, future research should replicate their study with more frequent measures to determine growth.

Implications for Current Practice

As a mathematics teacher in the secondary school in United States and South Korea, I was thinking about how to teach basic math skills to students with learning disabilities. They needed individualized instruction that contained a step-by-step process with much practice. All interventions that I reviewed in Chapter II were the perfect solution to apply to the mathematics lessons directly. These studies helped support the idea that technology including computers, calculators, and iPads, the number line strategy, and CRA strategy instruction are effective interventions to help not only elementary, but also secondary students with learning disabilities learn math in basic facts.

I will use these strategies in my future middle and high school math classes. The strategies were effective and helpful for students who needed special service, but there is no doubt that they work for students without disabilities.

Summary

The findings of these studies support that there are effective interventions to teach basic math to students with learning disabilities. When teachers implement one of the technical instruction tools, such as computer-based math fluency, calculators, iPads, the number line strategy, and CRA strategy, they should monitor students' growth, improvement, and performance in basic math. These efficient interventions will help students with learning disabilities increase not only their math abilities, but also their self-esteem.

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